SRI LANKA STANDARD

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SAMPLING PROCEDURES FOR INSPECTION BY VARIABLES

PART 1: SPECIFICATION FOR SINGLE SAMPLING PLANS INDEXED BY ACCEPTANCE QUALITY LIMIT (AQL) FOR LOT-BY-LOT INSPECTION FOR A SINGLE QUALITY CHARACTERISTIC AND A SINGLE AQL

SRI LANKA STANDARDS INSTITUTION

Sri Lanka Standard
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Gr. X

SRI LANKA STANDARDS INSTITUTION
No. 17, Victoria Place
Elvitigala Mawatha
Colombo 08
SRI LANKA

Sri Lanka Standards are subject to periodical revision in order to accommodate the progress made by industry. Suggestions for improvement will be recorded and brought to the notice of the Committees to which the revisions are entrusted.

This standard does not purport to include all the necessary provisions of a contract.

Sri Lanka Standard
SAMPLING PROCEDURES FOR INSPECTION BY VARIABLES – PART 1 : SPECIFICATION FOR SINGLE SAMPLING PLANS INDEXED BY ACCEPTANCE QUALITY LIMIT (AQL) FOR LOT-BY-LOT INSPECTION FOR A SINGLE QUALITY CHARACTERISTIC AND A SINGLE AQL

NATIONAL FOREWORD

This standard was approved by the Sectoral Committee on Building and Construction Materials and was authorized for adoption and publication as a Sri Lanka Standard by the Council of the Sri Lanka Standard Institution on 2016-07-22.

This Sri Lanka Standard is identical with **ISO 3951-1: 2013,** published by the International Organization for Standardization (**ISO**).

This Sri Lanka standard specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL).

TERMINOLOGY AND CONVENTIONS

The text of the International Standard has been accepted as suitable for publication as a Sri Lanka Standard. However, certain terminology and conventions are not identical with those used in Sri Lanka Standards.
Attention is therefore drawn to the following:

a) Wherever the "International Standard" appear referring to this standard they should be interpreted as "Sri Lanka Standard".
b) Wherever page numbers are quoted, they are "**ISO**" page numbers.
c) The coma has been used throughout as a decimal marker. In Sri Lanka Standards it is the current practice to use a full point on the base line as the decimal marker.

For the purpose of deciding whether a particular requirement of this standard is complied with, the final value, observed or calculated, expressing the result of a test method or observation shall be rounded off in accordance with **SLS 102**. The number of significant places retained in the rounded off value shall be the same as that of the specified value in this standard.

CROSS REFERENCES

International Standard	**Corresponding Sri Lanka Standard**
ISO 2859-1 : Sampling procedures for inspection by attributes -- Part 1 : Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection	SLS ISO 2859-1 : Sampling procedures for inspection by attributes -- Part 1 : Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection

SLS ISO 3951-1:2016

INTERNATIONAL STANDARD

ISO 3951-1

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Sampling procedures for inspection by variables —

Part 1:
Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

Règles d'échantillonnage pour les contrôles par mesures —

Partie 1: Spécification pour les plans d'échantillonnage simples indexés par un niveau de qualité acceptable (NQA) pour un contrôle lot par lot pour une caractéristique de qualité unique et un NQA unique

Reference number
ISO 3951-1:2013(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 69, *Application of statistical methods,* SC 5, *Acceptance sampling.*

This second edition cancels and replaces the first edition (ISO 3951-1:2006), of which it constitutes a minor revision with the following changes:

— procedures have been introduced to accommodate measurement uncertainty;

— many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

— *Part 4: Procedures for assessment of declared quality levels*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL) and is designed for users who have simple requirements. (A more comprehensive and technical treatment is given in ISO 3951-2.) This part of ISO 3951 is complementary to ISO 2859-1.

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

a) an automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;

b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is intended for application to a single quality characteristic that is measurable on a continuous scale. For two or more such quality characteristics, see ISO 3951-2.

It is assumed in the body of this part of ISO 3951 that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see Annex O, which was derived from Reference [20] in the Bibliography.

For double specification limits, this part of ISO 3951 treats combined control. For other types of control, refer to ISO 3951-2.

CAUTION — The procedures in this part of ISO 3951 **are not suitable for application to lots that have been screened for nonconforming items.**

Inspection by variables for percent nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

— unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;

— a single specification limit, or combined control of double specification limits;

— normal inspection, tightened inspection, or reduced inspection.

Table 1 is intended to facilitate the use this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. The table only deals with Clauses 15, 16, 20, 21, and 22; in every case, it is necessary, first of all, to have read the other clauses.

Table 1 — Summary table

Inspection	Single specification limit						Double specification limits with combined control					
	s–method			σ–method			s–method			σ–method		
	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts	Clauses or sub-clauses	Tables/ Annexes	Charts
Normal inspection	16.1, 16.2, 16.3, 21.1	A.1, B.1, B to R	B to R	17.1, 17.2, 21.1	A.1, C.1, B to R[a]	B to R[a]	16.1, 16.4, 21.1	A.1, D.1, F.1 (for n = 3), G.1 (for n = 3 or 4), B to R[a]	s-D to s-R, B to R[a]	17.1, 17.3 and 21.1	A.1, C.1, E.1, B to R[a]	B to R[a]
Switching between normal and tightened inspection	21.2, 21.3	B.1, B.2	B to R	21.2, 21.3	C.1, C.2	B to R[a]	21.2, 21.3	D.1, D.2	s-D to s-R, B to R[a]	21.2, 21.3	C.1, C.2, E.1	B to R[a]
Switching between normal and reduced inspection	21.4, 21.5	B.1, B.3	B to R	21.4, 21.5	C.1, C.3, I	B to R[a]	21.4, 21.5	D.1, D.3, G.1 (for n = 3 or 4)	s-D to s-R, B to R[a]	21.4, 21.5	C.1, C.3, E.1	B to R[a]
Switching between tightened and dis- contin- ued inspec- tion	22	B.2	B to R	22	C.2	B to R[a]	22	D.2	s-D to s-R, B to R[a]	22	E.1	B to R[a]
Switching between the s-method and σ-method	23	Annex J		23	Annex J		23	Annex J		23	Annex E, Annex J	

[a] But see 8.4.

Fifteen annexes are provided. Annexes A to I provide the tables needed to support the procedures. Annex J indicates how the sample standard deviation, s, and the presumed known value of the process standard deviation, σ, should be determined. Annex K provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection as well as for the s–method and σ–method. Annex L provides similar information for the producer's risks. Annex M gives the general formula for the operating characteristic of the σ–method. Annex N provides the statistical theory underlying the estimation of the process fraction nonconforming under the s–method for sample sizes 3 and 4, which, for technical reasons, are treated differently from the other sample sizes in this part of ISO 3951. Annex O provides procedures for accommodating measurement uncertainty.

Sampling procedures for inspection by variables —

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

1 Scope

This part of ISO 3951 is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process;

b) where only a single quality characteristic, x, of these products is taken into consideration, which must be measurable on a continuous scale;

c) where production is stable (under statistical control) and the quality characteristic, x, is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines a lower specification limit, L, an upper specification limit, U, or both; an item is qualified as conforming if and only if its measured quality characteristic, x, satisfies the appropriate one of the following inequalities:

1) $x \geq L$ (i.e. the lower specification limit is not violated);

2) $x \leq U$ (i.e. the upper specification limit is not violated);

3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are called cases with a single specification limit and 3), a case with double specification limits.

Where double specification limits apply, it is assumed in this part of ISO 3951 that conformance to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single AQL to the combined percentage of a product outside the two specification limits. This is referred to as combined control.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 2859-2, *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-2, *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1, and ISO 3534-2 and the following apply.

3.1
inspection by variables
inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2]

3.2
sampling inspection
inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2]

3.3
acceptance sampling inspection
acceptance sampling
sampling inspection (3.2) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6
acceptance quality limit
AQL
worst tolerable *process fraction nonconforming* (3.5) when a continuing series of lots is submitted for *acceptance sampling* (3.3)

Note 1 to entry: See Clause 5.

3.7
quality level
quality expressed as a rate of occurrence of nonconforming units

3.8
limiting quality
LQ
quality level (3.7), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* (3.3), is limited to a low probability of acceptance

[SOURCE: ISO 3534-2]

Note 1 to entry: See 14.1.

Note 2 to entry: In this part of ISO 3951: 10 %.

3.9
nonconformity
non-fulfilment of a requirement

3.10
nonconforming unit
unit with one or more nonconformities

[SOURCE: ISO 3534-2]

3.11
***s*–method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the sample standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 16.

3.12
***σ*–method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 17.

3.13
specification limit
conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2]

3.14
lower specification limit
L
specification limit (3.13) that defines the lower conformance boundary

[SOURCE: ISO 3534-2]

3.15
upper specification limit
U
specification limit (3.13) that defines the upper conformance boundary

[SOURCE: ISO 3534-2]

3.16
combined control
requirement when both upper and lower limits are specified for the quality characteristic and an *AQL* (3.6) that applies to the combined percent nonconforming beyond the two limits is given

Note 1 to entry: See 5.3.

Note 2 to entry: The use of combined control implies that nonconformity beyond either *specification limit* (3.13) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.17
acceptability constant
k
constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.2 and 17.2.

3.18
quality statistic
Q
function of the *specification limit* (3.13), the sample mean, and the sample or process standard deviation used in assessing the acceptability of a lot

[SOURCE: ISO 3534-2]

Note 1 to entry: For the case of a single *specification limit* (3.13), the lot may be sentenced on the result of comparing *Q* with the *acceptability constant* (3.17)*k*.

Note 2 to entry: See 16.2 and 17.2.

3.19
lower quality statistic
Q_L
function of the *lower specification limit* (3.14), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *lower specification limit* (3.14), the lot is sentenced on the result of comparing Q_L with the *acceptability constant* (3.17)*k*.

[SOURCE: ISO 3534-2]

Note 2 to entry: See Clause 4, 16.2, and 17.2.

3.20
upper quality statistic
Q_U
function of the *upper specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *upper specification limit* (3.15), the lot is sentenced on the result of comparing Q_U with the *acceptability constant* (3.17)*k*.

[SOURCE: ISO 3534-2]

Note 2 to entry: See Clause 4, 16.2, and 17.2.

3.21
maximum sample standard deviation
MSSD
s_{max}
largest sample standard deviation for a given sample size code letter, inspection severity, and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criteria for the combined control of double *specification limits* (3.13) when the process variability is unknown

Note 1 to entry: See 16.4.

3.22
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double specification limits with a combined *AQL* (3.6) requirement under tightened inspection with known process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 17.3.

3.23
switching rule
instruction within an *acceptance sampling* (3.3) scheme for changing from one *acceptance sampling* (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[SOURCE: ISO 3534-2]

Note 1 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of 'severity'.

Note 2 to entry: See Clause 21.

3.24
measurement
set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2]

4 Symbols

The symbols used are as follows:

c_U factor for determining the upper control limit for the sample standard deviation (See Annex H.)

f_s factor that relates the maximum sample standard deviation to the difference between U and L (See Annex D)

f_σ factor that relates the maximum process standard deviation under tightened inspection to the difference between U and L (See Annex E)

k Form k acceptability constant for use with a single quality characteristic and a single specification limit (See Annex B for the s–method or Annex C for the σ–method)

L lower specification limit (As a subscript to a variable, it denotes its value at L.)

m process mean

N lot size (number of items in a lot)

n sample size (number of items in a sample)

$\hat{p}$ estimate of the process fraction nonconforming

$\hat{p}_L$ estimate of the process fraction nonconforming below the lower specification limit

$\hat{p}_U$ estimate of the process fraction nonconforming above the upper specification limit

p^* maximum acceptable value for the estimate of the process fraction nonconforming

P_a probability of acceptance

Q quality statistic

Q_L lower quality statistic

NOTE Q_L is defined as $(\bar{x}-L)/s$ when the process standard deviation is unknown, and as $(\bar{x}-L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE Q_U is defined as $(U-\bar{x})/s$ when the process standard deviation is unknown, and as $(U-\bar{x})/\sigma$ when it is presumed to be known.

s sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s=\sqrt{\frac{\sum_{j=1}^{n}(x_j-\bar{x})^2}{n-1}}$$

(See Annex J.)

s_{max} maximum sample standard deviation (MSSD)

σ standard deviation of a process that is under statistical control

NOTE σ^2, the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD)

U upper specification limit (As a suffix to a variable, it denotes its value at U.)

x_j measured value of the quality characteristic for the j^{th} item of the sample

$\bar{x}$ the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x}=\frac{\sum_{j=1}^{n}x_j}{n}$$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the AQL may be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification or in the contract, or by the responsible authority. Where both upper and lower specification limits are given, this part of ISO 3951 addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the two limits; this is known as “combined control.” (See ISO 3951-2 for “separate” and “complex” control of double specification limits.)

5.4 Preferred AQLs

The 16 AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. They are only preferred in the sense that they are the AQL values used in the tabulations and charts. It follows that, if for any product or service, an AQL other than a preferred AQL is designated, then this part of ISO 3951 is not applicable. (See 14.2.)

5.5 Caution

From the definition of the AQL in 5.1, it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

6 Switching rules for normal, tightened, and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral and, therefore, obligatory procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see 20.1) that the variability is in statistical control, consideration should be given to switching to the σ–method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see Clause 23).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses 21, 22, and 23.

7 Relation to ISO 2859-1

7.1 Similarities

The similarities are as follows.

a) This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans, and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).

c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. General tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the s–method and σ–method, and for normal, tightened, and reduced inspection.

d) The switching rules are essentially equivalent.

7.2 Differences

a) **Determination of acceptability.** Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this part of ISO 3951, two methods are considered: the s–method, for use when the process standard deviation, s, is unknown, and the σ–method, for use when s is presumed to be known. In the case of a single specification limit, the acceptability may be calculated from a formula (see 16.2 and 17.2), but for the s–method, it is also easily established by a graphical method (see 16.3). In the case of combined control of double specification limits under the s–method, this part of ISO 3951 provides only for a graphical method of determining acceptability (see 16.4); for combined control of double specification limits under the σ–method, a numerical method is given.

b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution or a close approximation to a normal distribution.

c) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves for unknown process standard deviation have been matched by minimizing the area between the curves representing the squares of the OC values, a method that gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that for most practical purposes, the attributes and variables OC curves may be considered to be identical. The plans for known process standard deviation were derived by minimizing the area between the squared OC functions subject to keeping the same form p^* acceptability constant as for the corresponding case for unknown process standard deviation, i.e. only the sample size was open to choice, so the match was, in general, less perfect.

d) **Producer's risk**. For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1.

NOTE The producer's risks of the plans are given in Annex L.

e) **Sample sizes.** The variables sample sizes for given combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes. This is particularly true for the *σ*–method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.

f) **Double sampling plans.** Double sampling plans by variables are presented separately, in ISO 3951-3.

g) **Multiple sampling plans.** No multiple sampling plans by variables are given in this part of ISO 3951.

h) **Average outgoing quality limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

8 Consumer protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal, and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Some users may select specific individual plans from this part of ISO 3951 and use them without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as "inspection in compliance with ISO 3951-1." When used in such a way, ISO 3951-1 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

8.2 Consumer's risk quality (CRQ) tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give a consumer's risk quality (CRQ) not worse than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer's risk quality and a consumer's risk to be associated with it. Annex K gives values of consumer's risk quality for the *s*–method and *σ*–method corresponding to a consumer's risk of 10 %.

However, application of this part of ISO 3951 to isolated lots is deprecated, as the theory of sampling by variables applies to a process. For isolated lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2. (See also Reference [5] in the Bibliography.)

8.3 Producer's risk tables

Annex L gives the probability of non-acceptance under the *s*–method and *σ*–method for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer's risk.

8.4 Operating characteristic (OC) curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic curve. OC curves for the normal inspection *s*–method sampling plans of this part of ISO 3951 are given in Charts B to R, which should be consulted when choosing a sampling plan. Also given are tables of process qualities at nine standard probabilities of acceptance for all of the *s*–method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the *s*–method. Most of them also provide a good approximation to the *σ*–method and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the σ–method, refer to Annex M.

9 Allowing for measurement uncertainty

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic, X, of the items in the lots is normally distributed with unknown process mean, μ, and either known or unknown process standard deviation σ. The assumption is also made that X can be measured without measurement error, i.e. that measurement of an item with the true value, x_i, results in the value x_i. However, the master tables can also be used, with appropriate adjustments, in the presence of measurement error.

If the measurement standard deviation is no greater than 10 % of the process standard deviation, it can be ignored. For measurement standard deviation greater than 10 % of the process standard deviation, the sample size will need to be increased, although the acceptability constant remains the same. Moreover, if neither the measurement standard deviation nor the process standard deviation is known, more than one measurement will need to be made on each sampled item and the total variability of the measurements will need to be separated into the components due to the measurements and to the process.

Details are provided in Annex O.

10 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge both of statistics and the product to be inspected. Clauses 11 to 13 of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

11 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely how good the product is. Earlier warning may, therefore, be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in 16.4.2 and 16.4.4.)

d) From a comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes, such as from ISO 2859-1, and the standard plans in this part of ISO 3951, it will be seen that the smallest samples are generally required by the σ–method (used when the process standard deviation is presumed to be known). The sample sizes for the s–method (used when the process standard deviation is unknown) are also, in general, substantially smaller than for sampling by attributes.

e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases. (For two or more quality characteristics, this part of ISO 3951 does not apply. See ISO 3951-2 for details.)

h) The use of this part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of the quality characteristic is normal. In case of doubt, the responsible authority should be consulted.

NOTE 1 Departure from normality may be caused by the presence of outliers, the assessment and accommodation of which is considered in ISO 16269-4.

NOTE 2 ISO 5479 gives detailed procedures for tests for departure from normality.

12 Choice between the *s*-method and *σ*-method

If it is desired to apply inspection by variables, the next question is whether to use the s–method or the σ–method. The σ–method is the most economical in terms of sample size, but before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the s–method, but subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, whether it will be economical to change to the σ–method. The size of the sample will generally be smaller and the acceptability criterion simpler under the σ–method. (See 17.2) On the other hand, it will still be necessary to calculate the sample standard deviation, s, for record purposes and to keep the control charts up to date. (See Clause 20.) The calculation of s can appear daunting at first sight, but the difficulty is more apparent than real; this is especially true if a calculator or computer is available. Methods of determining s and σ are given in Annex J.

13 Choice of inspection level and AQL

For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate OC curve from Charts B to R or the appropriate table from Tables B to R shows the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

Three inspection levels, I, II, and III, are given in Table 1 for general use. The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate. Level I may be used when less discrimination is needed or level III when greater discrimination is required. Four additional special levels, S-1, S-2, S-3, and S-4, are also given in Table 1 and may be used when relatively small sample sizes are necessary and larger sampling risks can be tolerated.

14 Choice of sampling scheme

14.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the s–method, has been found in practice to produce workable sampling plans; but it rests on the assumption that the order of priority is first the AQL, second the sample size, and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 21, 22, and 23), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It should also be remembered that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

If, in certain circumstances, the limiting quality has a higher priority than the sample size, a suitable plan in this part of ISO 3951 may be selected by using Chart A. Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan that meets the specified requirements. This should be verified by inspecting the OC curve from among Charts B to R relating to this code letter and AQL.

However, use of this method is deprecated for isolated lots or short series of lots. (See 8.2.)

EXAMPLE Suppose that an acceptable value for the limiting quality is 6,0 % nonconforming and that the desired quality with a 95 % probability of acceptance is 2,0 % nonconforming. A vertical line on Chart A at 6,0 % nonconforming and a horizontal line at 2,0 % nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with sample size code letter L and AQL 1,5 % meets the requirements.

If the horizontal and vertical lines intersect at a point above the line marked R in Chart A, this implies that the specification cannot be met by any of the plans in this part of ISO 3951.

14.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

15 Preliminary operations

Before starting inspection by variables:

a) check that production is considered to be continuing and that the distribution of the quality characteristic can be considered to be normal;

   NOTE 1 For tests for departure from normality, see, for example, ISO 16269-3.

   NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

b) check whether the s–method is to be used initially or whether the standard deviation is stable and known, in which case the σ–method should be used;

c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) check, for a quality characteristic with double specification limits, that nonconformities beyond each limit are of equal importance. If this is not the case, refer to ISO 3951-2;

e) check that an AQL has been designated, and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

16 Standard procedures for the *s*-method

16.1 Obtaining a plan, sampling, and preliminary calculations

The procedure for obtaining and implementing a plan is as follows:

a) With the inspection level given (normally this will be level II) and with the lot size, obtain the sample size code letter using Table A.1.

b) For a single specification limit, enter Table B.1, B.2, or B.3 as appropriate with this code letter and the AQL and obtain the sample size, n, and the acceptability constant, k. For combined control of double specification limits when the sample size is 5 or more, find the appropriate acceptance curve from among Charts *s*-D to *s*-R.

c) Take a random sample of size n, measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean, and s, the sample standard deviation (see Annex J). If $\bar{x}$ lies outside the specification limit(s), the lot can be judged unacceptable without even calculating s. It is, however, necessary to calculate s for record purposes.

16.2 Acceptability criteria for single specification limits

If single specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s} \tag{1}$$

or

$$Q_L = \frac{\bar{x} - L}{s} \tag{2}$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the acceptability constant, k, obtained from Table B.1, B.2, or B.3 for normal, tightened, or reduced inspection, respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if it is less, the lot is not acceptable.

Thus, if only the upper specification limit, U, is given, the lot is

acceptable if $Q_U \geq k$, and

not acceptable if $Q_U \leq k$,

or, if only the lower specification limit, L, is given, the lot is

acceptable if $Q_L \geq k$, and

not acceptable if $Q_L \leq k$.

EXAMPLE 1 Single upper specification limit.

The maximum temperature of operation for a certain device is specified as 60 °C and the operating temperature is known from previous experience to be normally distributed. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL of 2,5 % is to be used. From Table A.1, the sample size code letter is found to be F; from Table B.1, it is seen that a sample size of 13 is required and that the acceptability constant, k, is 1,426. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	54,62 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See K.1.2)	3,330 °C
Specification limit (upper): U	60 °C
Upper quality statistic: $Q_U = (U - \bar{x}) / s$	1,617
Form k acceptability constant: k (from Table B.1)	1,426
Acceptability criterion: Is $Q_U \geq k$?	Yes (1,617 > 1,426)

The lot meets the acceptability criterion and is therefore acceptable.

EXAMPLE 2 Single, lower specification limit, requiring the use of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From Table A.1, it is seen that the sample size code letter is J. However, on entering Table B.1 with sample size code letter J and AQL of 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample size 28 and acceptability constant k = 2,580. A random sample of size 28 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,40	6,44	6,34	6,04	6,15	6,29	6,63
6,44	7,15	6,70	6,59	6,51	6,80	5,94	6,35	7,17	6,83	6,25	6,96	7,00	6,38

Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	28
Sample mean: $\bar{x} = \sum x / n$	6,551 s
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See K.1.2)	0,325 1 s
Lower specification limit: L	4,0 s
Lower quality statistic: $Q_L = (\bar{x} - L) / s$	7,847
Form k acceptability constant: k (from Table B.1)	2,580
Acceptability criterion: Is $Q_L \geq k$?	Yes (7,847 > 2,580)

The lot meets the acceptability criterion, so it is acceptable.

16.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$$\bar{x} = U - ks \text{ (for an upper limit)}$$

or

$$\bar{x} = L + ks \text{ (for a lower limit)}$$

as appropriate on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit, the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Plot the point (s, $\bar{x}$) on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE Graphical method for upper specification limit.

Using the data given in EXAMPLE 1 of 15.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$ [As $k = 1{,}426$, this means that the line passes through the points ($s = 1$, $\bar{x} = 58{,}574$), ($s = 2$, $\bar{x} = 57{,}148$), ($s = 3$, $\bar{x} = 55{,}722$), etc.]. Select a suitable point and draw a straight line through both it and ($s = 0$, $\bar{x} = 60$), i.e. (0, U). The accept zone is then the area under this line.

The calculated values of s and $\bar{x}$ are 3,330 and 54,62. The point (s, $\bar{x}$) is seen from Figure 1 to lie within the accept zone; the lot is therefore acceptable.

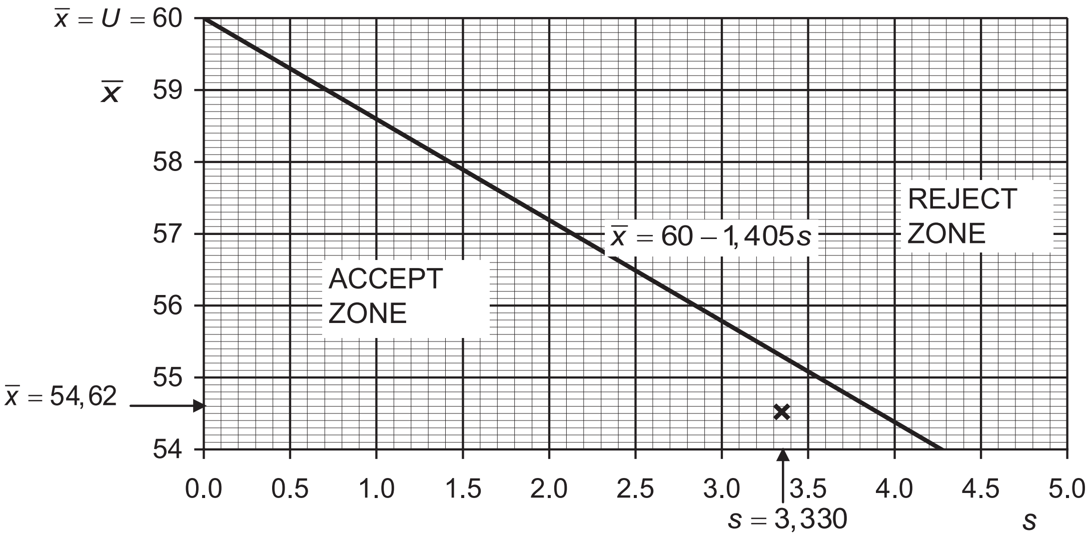


Figure 1 — Example of the use of an acceptance chart for a single specification limit, s—method

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot, the point (s, $\bar{x}$) may be plotted to determine whether or not the lot is acceptable.

16.4 Acceptability criterion for combined control of double specification limits

16.4.1 General

For the s–method with combined control of the upper and lower specification limits, i.e. with an overall AQL for the percentage of items from the process outside the specification limits, this part of ISO 3951 provides a graphical method for determining lot acceptability for all sample sizes except for sample sizes 3 and 4. (ISO 3951-2 provides numerical methods only.) The larger the sample variability, the less likely it is that the requirement is satisfied. If the value of s exceeds the value of the maximum sample standard deviation (MSSD) obtained from Table D.1, D.2, or D.3, no further calculation or reference to graphs is required, for the lot shall be immediately judged unacceptable.

Numerical methods are provided for the combined control of double specification limits for sample sizes 3 and 4.

16.4.2 Procedure for sample size 3

It may be seen from Annex B that there are three cases where the required sample size is 3 for the *s*–method, i.e. for sample size code letter B under normal inspection for an AQL of 4 %, for sample size code letter B under tightened inspection for an AQL of 6,5 %, and for sample size code letter range B to D under reduced inspection for an AQL of 1,5 %.

For sample size 3, proceed as follows. After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s, find the applicable value of the coefficient f_S from Table D.1, D.2, or D.3. Determine the MSSD (i.e. the maximum allowable) from Formula (3).

$$MSSD = s_{max} = (U - L)f_S \tag{3}$$

Then compare s with s_{max}. If s is greater than s_{max}, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x}) / s$ and $Q_L = (\bar{x} - L) / s$. Multiply Q_U and Q_L by $\sqrt{3}/2$ (i.e. approximately 0,866) and use Table F.1 to determine the estimates $\hat{p}_U$ and $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and lower limits, respectively.

NOTE 1 Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0,5 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this part of ISO 3951. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming may be obtained by entering Table F.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. E.g. if $Q_U = -0{,}156$, then $\sqrt{3}Q_U/2 = -0{,}135$; entering Table F.1 with 0,135 gives an estimate of 0,456 9; subtracting this from 1,0 gives $\hat{p}_U = 0{,}543\,1$.

NOTE 2 The basis of Table F.1 is given in Annex K. Instead of using Table F.1, the estimate of the process fraction nonconforming beyond each specification limit when $n = 3$ may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3} \\ \dfrac{2}{\pi} arc\sin\left[\sqrt{(1 - Q\sqrt{3}/2)/2}\right] & \text{if } -2/\sqrt{3} \le Q \le 2/\sqrt{3} \\ 1 & \text{if } Q < -2/\sqrt{3} \end{cases} \tag{4}$$

These two estimates must be added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^* given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 3.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so combined control of the double specification limits is appropriate. The specification limits are set at 10 m either side of the point of aim at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table A.1, the sample size code letter is found to be B. From Table B.1, it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding errors −5,0 m, 6,7 m, and 8,8 m. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Value obtained**
Sample size: n	3
Sample mean: $\bar{x}=\frac{1}{n}\sum_{j=1}^{n} x_j$	3,5 m
Sample standard deviation: $s=\sqrt{\sum_{j=1}^{n}(x_j-\bar{x})^2/(n-1)}$ (See J.1.2.)	7,436 m
Value of f_s for MSSD (Table D.1)	0,475
MSSD, $s_{max} = (U - L)f_s = [10 - (-10)] \times 0{,}475$	9,50

Since $s = 7{,}436 < s_{max} = 9{,}50$, the lot may be acceptable, so continue with the calculations.

$Q_U = (U-\bar{x}) / s = (10 - 3{,}5) / 7{,}436$	0,874 1
$Q_L = (\bar{x}-L) / s = (3{,}5 + 10) / 7{,}436$	1,815
$\sqrt{3}Q_U/2$	0,757
$\sqrt{3}Q_L/2$	1,572
$\hat{p}_U$	0,226 7
$\hat{p}_L$ (from Table F.1)	0,000 0
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,2267
p^* (from Table G.1, normal inspection)	0,1924

Since $\hat{p} > p^*$ the lot is not acceptable.

NOTE This lot is not acceptable even though all inspected items in the sample are within the specification limits.

16.4.3 Procedure for sample size 4

For sample size 4 under the s–method, proceed as follows. After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s, find the applicable value of the coefficient f_s from Table D.1, D.2, or D.3. Determine the MSSD (i.e. the maximum allowable) from Formula (5)

$$\mathrm{MSSD} = s_{max} = (U - L)f_s \tag{5}$$

Then, compare s with the MSSD. If s exceeds the MSSD, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x}) / s$ and $Q_L = (\bar{x} - L) / s$. Calculate

$$\hat{p}_U = \begin{cases} 1 & \text{if } Q_U \leq -1{,}5 \\ 0{,}5 - Q_U / 3 & \text{if } -1{,}5 < Q_U < 1{,}5 \\ 0 & \text{if } Q_U \geq 1{,}5 \end{cases} \tag{6}$$

and

$$\hat{p}_L = \begin{cases} 1 & \text{if } Q_L \leq -1{,}5 \\ 0{,}5 - Q_L / 3 & \text{if } -1{,}5 < Q_L < 1{,}5 \\ 0 & \text{if } Q_L \geq 1{,}5 \end{cases} \tag{7}$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

NOTE The basis of Formulae (6) and (7) is given in Annex N.

EXAMPLE Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming using an AQL of 2,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table A.1, the sample size code letter is found to be C. From Table B.1, it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters of 82,4 mm, 82,2 mm, 83,1 mm, and 82,3 mm. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Value obtained**
Sample size: n	4
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	82,50 mm
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n} (x_j - \bar{x})^2 / (n-1)}$ (See Annex J, J.1.2.)	0,408 2 mm
Upper specification limit: U	84,0 mm
Lower specification limit: L	82,0 mm
Value of f_s for MSSD (Table D.1)	0,365
$s_{max} = (U - L) f_s = (84 - 82) \times 0{,}365$	0,730 mm

Since s = 0,408 2 < s_{max} = 0,730, the lot may be acceptable, so continue with the calculations.

Further information needed	**Value obtained**
$Q_U = (U - \bar{x}) / s = (84 - 82,5) / 0,408\ 2$	3,674 7
$Q_L = (\bar{x} - L) / s = (82,5 - 82)/0,408\ 2$	1,224 9
$\hat{p}_U$ [from Formula (6) above]	0,000 0
$\hat{p}_L$ [from Formula (7) above]	0,091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,091 7
p^* (from Table G.1, normal inspection)	0,086 0

Since $\hat{p} > p^*$, the lot is not acceptable.

16.4.4 Procedure for sample sizes greater than 4

After calculating the sample mean, $\bar{x}$, and the sample standard deviation, s, find the applicable value of the coefficient f_s from Table D.1, D.2, or D.3. Determine the MSSD (i.e. the maximum allowable) from the formula

$$\text{MSSD} = s_{max} = (U - L) f_s$$

Compare s with s_{max}. If s is greater than s_{max}, then the lot may be rejected without further calculation.

Otherwise, from among Charts s-D to s-R, consult the chart labelled with the appropriate sample size code letter and select the acceptance curve with the AQL specified for the two limits.

Calculate the values of $s / (U - L)$ and $(\bar{x} - L) / (U - L)$ and plot a point representing these values on a copy of the graph. If the point lies inside the curve, the lot is acceptable; if it lies outside, the lot is not acceptable.

It is recommended that before the inspection operations begin, the required acceptance curves for normal, tightened, and reduced inspection be copied. The scales should be adjusted so that s and $\bar{x}$ can be plotted directly (i.e. the upper specification limit is given instead of 1,0 and the lower specification limit instead of 0,0 on the vertical scale).

Plot a point on the chart representing the values of s and $\bar{x}$ found from the sample. If the point lies inside the curve, the lot is acceptable; if it lies outside, the lot is not acceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 5 or more.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL of 1,5 %, is to be used. From Table A.1, the sample size code letter is found to be E; from Table B.1, it is seen that a sample of 13 is required, and from Table D.1, that the value of f_s for the MSSD under normal inspection is 0,274. Suppose the measurements obtained are as follows: 63,5 °C; 61,9 °C; 65,2 °C; 61,7 °C; 68,4 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	64,223 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$	2,789 9 °C
(See K.1.2)	
Upper specification limit: U	70,0 °C
Lower specification limit: L	60,0 °C
Value of f_s for MSSD (Table D.1 for normal inspection)	0,274
MSSD = $s_{max} = (U - L) f_s = (70 - 60) \times 0{,}274$	2,74 °C

Since the value of s exceeds s_{max}, the lot may immediately be adjudged unacceptable.

NOTE This lot is not acceptable even though all inspected items in the sample are within the specification limits.

Suppose that the AQL had been 2,5 % instead of 1,5 %. In this case, the value of f_s would be 0,285, so s_{max} is equal to (70 − 60) × 0,285 = 2,85 °C. Since s is now less than s_{max}, it is not possible to determine immediately whether or not the lot is acceptable.

The appropriate acceptance curve is taken from Chart s-E. If the scales have been adjusted to the real measurements, plot the point (s = 2,79, $\bar{x}$ =64,22), as on Figure 2. The point lies just outside the acceptance curve for an AQL of 2,5 % so the lot is not acceptable.

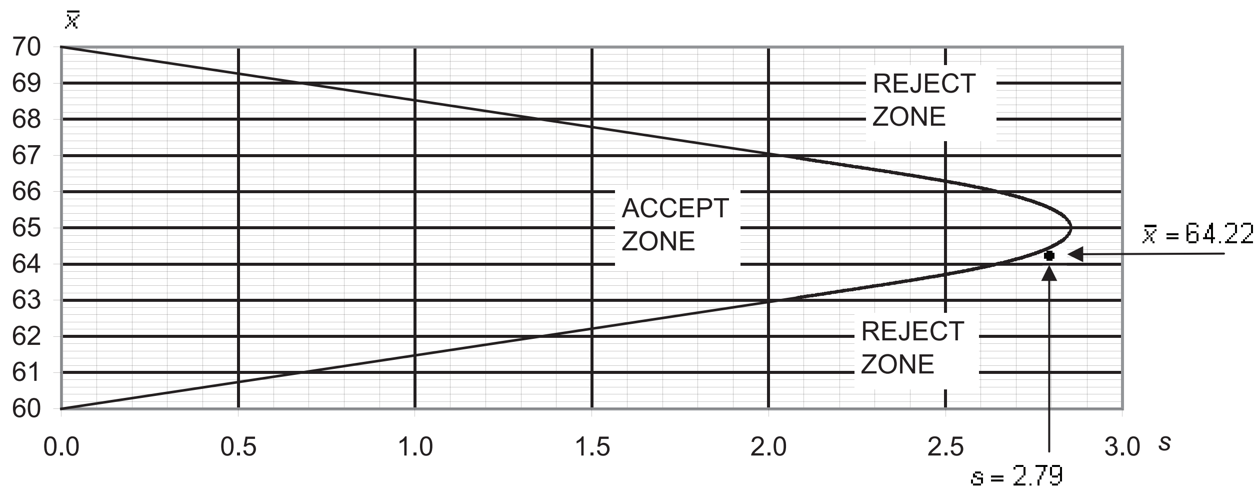


Figure 2 — Example of the use of an acceptance chart for combined control of double specification limits: *s*-method with actual scales

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required:

a) standardized sample mean: ($\bar{x}$ − L) / (U − L) = (64,22 − 60) / (70 − 60) = 0,422;

b) standardized sample standard deviation: s / (U − L) = 2,79/(70 − 60) = 0,279.

The point (0,279, 0,422) is plotted on Figure 3. Since it lies outside the acceptance curve for AQL of 2,5 %, the lot is not acceptable.

17 Standard procedures for the σ-method

17.1 Obtaining a plan, sampling, and preliminary calculations

The σ–method shall only be used when there is valid evidence that the standard deviation σ of the process can be considered constant with a known value.

From Table A.1, obtain the sample size code letter. Then, depending on the severity of inspection, enter Table C.1, C.2, or C.3 with the sample size code letter and the specified AQL to obtain the sample size, *n*, and acceptability constant, *k*.

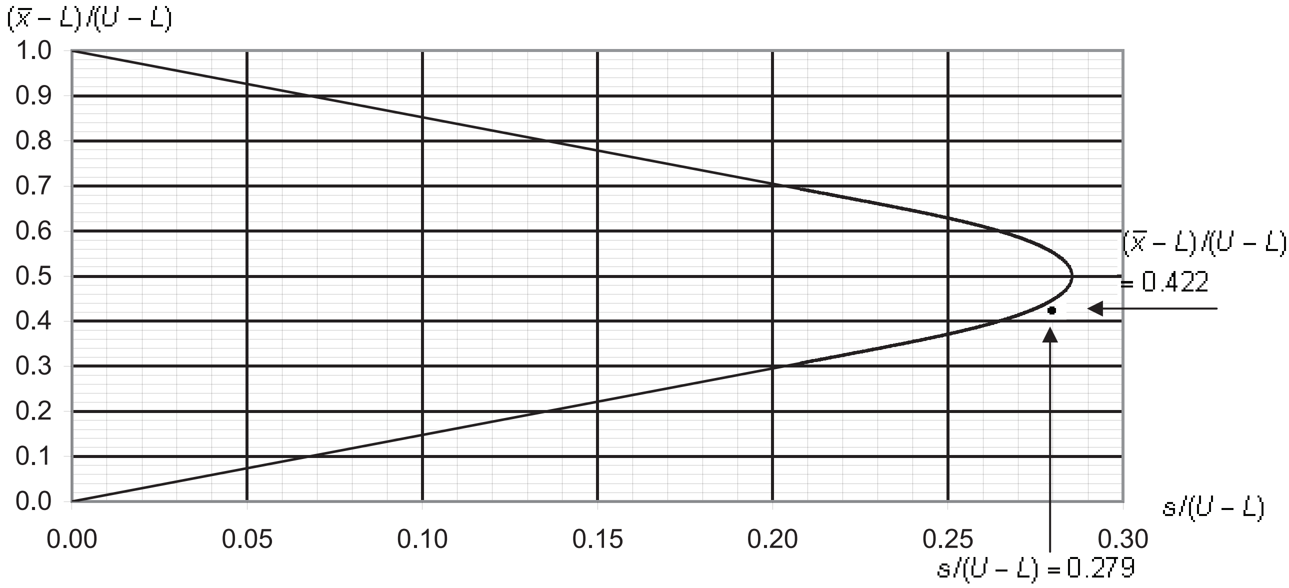


Figure 3 — Example of the use of an acceptance chart for combined control of double specification limits: *s*-method with normalized scales

Take a random sample of this size, measure the characteristic under inspection, *x*, for all items of the sample, and calculate the sample mean, $\bar{x}$. (The sample standard deviation, *s*, should also be calculated but only for the purpose of checking the continued stability of the process standard deviation. See Clause 20.)

17.2 Acceptability criteria for a single specification limit

The acceptability criterion can be found by following a procedure similar to the one given for the *s*–method.

First, replace the *s* derived from the individual samples by σ, the presumed known value of the standard deviation of the process, and then compare the calculated value of *Q* with the value of the acceptability constant, *k*, obtained from one of the Tables C.1, C.2, and C.3.

Note, for example, that the acceptability criterion Q_U [$= (U - \bar{x}) / \sigma$] $\geq k$ for an upper specification may be written as $\bar{x} \leq U - k\sigma$. Since *U*, *k*, and σ are all known in advance, the acceptance value $\bar{x}_U$ [$= U - k\sigma$] should therefore be determined before the inspection begins.

For an upper specification limit, a lot will be

acceptable if $\bar{x} \leq \bar{x}_U$ [$= U - k\sigma$], and

not acceptable if $\bar{x} > \bar{x}_U$ [$= U - k\sigma$].

Similarly, for a lower specification limit, a lot will be

acceptable if $\bar{x} \geq \bar{x}_L$ [$= L + k\sigma$], and not acceptable if $\bar{x} < \bar{x}_L$ [$= L + k\sigma$].

EXAMPLE Determination of acceptability for a single specification limit using the σ–method.

The specified minimum yield point for certain steel castings is 400 N/mm^2. The next lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL of 0,65 %, is to be used. The value of σ is considered to be 21 N/mm^2. From Table A.1, it is seen that the sample size code letter is H. Then, from Table C.1, it is seen that for an AQL of 1,0 %, the sample size, n, is 11 and the acceptability constant, k, is 2,046. Suppose the yield points of the sample specimens are: 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400. Compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Acceptability constant: k	2,046
Known σ	21 N/mm^2
Product: $k\,\sigma$	42,97 N/mm^2
Specification limit: L	400 N/mm^2
Acceptance value: $\bar{x}_L = L + k\sigma$	442,97 N/mm^2
Sum of measurement results: $\sum x$	471 3 N/mm^2
Sample size: n	11
Sample mean: $\bar{x}$	428,5 N/mm^2
Acceptability criterion: Is $\bar{x} \geq \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion; therefore, the lot is not acceptable.

17.3 Acceptability criterion for combined control of double specification limits

For combined control of the upper and the lower specification limits, i.e. where there is an overall AQL for the percentage of the process outside the specification limits, the following approximative procedure may be used.

a) Before sampling, enter Table E.1 with the AQL to determine the value of the factor f_σ.

b) Calculate the maximum allowable value of the process standard deviation using the formula $\sigma_{max} = (U - L)f_\sigma$ for the MPSD.

c) Compare the value of the process standard deviation, σ, with σ_{max}. If σ exceeds σ_{max}, the process is unacceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

d) If $\sigma \leq \sigma_{max}$, then use the lot size and given inspection level to determine the sample size code letter from Table A.1.

e) From the sample size code letter and inspection severity (i.e. whether inspection is normal, tightened, or reduced), determine the sample size, n, and acceptability constant, k, from Table C.1, C.2, or C.3.

f) Calculate the upper allowable bound, $\bar{x}_U$, to sample means from the formula $\bar{x}_U = U - k\sigma$, and the lower allowable bound, $\bar{x}_L$, from the formula $\bar{x}_L = L + k\sigma$.

g) Select a random sample of size n from the lot and calculate the sample mean, $\bar{x}$. The acceptability criterion is: If $\bar{x}_L \leq \bar{x} \leq \bar{x}_U$, the lot is acceptable; if $\bar{x} < \bar{x}_L$ or $\bar{x} > \bar{x}_U$, the lot is not acceptable.

NOTE If $\bar{x}_L \le \bar{x} \le \bar{x}_U$ but $\sigma > 0{,}75\sigma_{max}$ and $\bar{x}$ is close to either $\bar{x}_L$ or $\bar{x}_U$, the exact method given in 17.3 of ISO 3951-2 shall be preferred.

EXAMPLE Determination of acceptability for combined control under the σ–method.

The specification for electrical resistance of a certain electrical component is (520 ± 50) Ω. Production is at a rate of 1 000 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 1,5 %, is to be used for the two specification limits (470 Ω and 570 Ω). σ is known to be 18,5 Ω.

Information needed	**Value obtained**
Factor from Table E.1: f_σ	0,194
Upper specification limit: U	570 Ω
Lower specification limit: L	470 Ω
Maximum process standard deviation, $\sigma_{max} = (U - L)f_\sigma$	19,4 Ω
Known σ	18,5 Ω

Since σ is less than σ_{max}, the sample is analysed further for lot acceptability.

Entering Table A.1 with the lot size and inspection level, it is found that the sample size code letter is J; from Table C.1, it is seen that a sample size of 19 is required under normal inspection, with a Form k acceptance constant of 1,677. Suppose that the 19 sample values of the resistance, in Ω, are as follows: 515; 491; 479; 513; 521; 536; 483; 509; 514; 507; 484; 526; 532; 499; 530; 512; 492; 522; 488. Lot acceptability is to be determined.

Further information needed	**Value obtained**
Sample size: n (from Table C.1)	19
Acceptability constant (from Table C.1)	1,677
Upper bound for $\bar{x}$: $\bar{x}_U = U - k\sigma$	538,9 Ω
Lower bound for $\bar{x}$: $\bar{x}_L = L + k\sigma$	501,1 Ω
Sum of measurement results: $\sum x$	10,160 Ω
Sample mean: $\bar{x}$	508,0 Ω

Since the $\bar{x}$ at 508,0 Ω lies between the acceptance limits for $\bar{x}$ of 501,1 Ω and 538,9 Ω, the lot is acceptable.

NOTE 1 Although $\sigma > 0{,}75\sigma_{max}$, $\bar{x}$ is close to neither $\bar{x}_L$ nor $\bar{x}_U$, so the approximative method is suitable.

NOTE 2 All the calculations other than the last two lines should be completed before sampling begins.

NOTE 3 If, for example, σ had been known to be, say, 20, then σ exceeds the MPSD and, therefore, sampling inspection should not even have taken place.

18 Procedure during continuing inspection

As a variables sampling inspection plan can only operate efficiently if

a) the characteristic being inspected is normally distributed,

b) records are kept, and

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

19 Normality and outliers

19.1 Normality

The responsible authority should have checked for normality before sampling began. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables or use should be made of the tests for departure from normality such as those given in ISO 5479 or in Clause 2 of ISO 5725-2.

19.2 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, will produce an increase in variability, change the mean, and may consequently lead to non-acceptance of the lot. (See ISO 16269-4.) When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and the vendee.

20 Records

20.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and a warning given before an unacceptable standard is reached. This is only possible if adequate records are kept.

Whatever the method used, s–method or σ–method, records should be kept of the values of $\bar{x}$ and s, preferably in the form of control charts. (See, for example, ISO 7870 and ISO 8258.)

This procedure should be applied especially with the σ–method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ.

For combined control of double specification limits, the values of the MSSD, given in Table D.1, D.2, or D.3, should be plotted on the s control chart, as an indication of an unacceptable value.

NOTE Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedures given in Clauses 15 and 16.

20.2 Lots that are not accepted

Particular care shall be taken to record all lots that are not accepted and to see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

21 Operation of switching rules

The standard switching rules are as follows.

21.1 Normal inspection is used at the start of inspection (unless otherwise designated). It will be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

21.2 Tightened inspection shall be instituted when two lots on original normal inspection are not accepted within any five or fewer successive lots.

Tightened inspection is generally achieved by increasing the value of the acceptability constant k. The values are tabulated in Table B.2 for the s–method and Table C.2 for the σ–method.

21.3 Tightened inspection shall be relaxed when five successive lots on original inspection have been accepted on tightened inspection; then, normal inspection shall be reinstated.

21.4 Reduced inspection may be instituted after 10 successive lots have been accepted under normal inspection, provided that

— these lots would have been acceptable if the AQL had been one step tighter (e.g. 0,65 % instead of 1,0 %),

NOTE If a value of k for this tighter AQL is not given in Table B.1 (s–method) or Table C.1 (σ–method), refer to the supplementary acceptance constants provided in Table I.1.

— production is in statistical control, and

— reduced inspection is considered desirable by the responsible authority.

Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of n and k for reduced inspection are given in Table B.3 for the s–method and Table C.3 for the σ–method.

When the previous 10 lots have been accepted under original inspection, reduced inspection may be instituted without the condition that these lots would have been acceptable if the AQL had been one step tighter, subject to the approval of the responsible authority.

21.5 Reduced inspection shall cease and normal inspection be reinstated if any of the following occur on original inspection:

a) a lot is not accepted;

b) production becomes irregular or delayed;

c) reduced inspection is no longer considered desirable by the responsible authority.

22 Discontinuation and resumption of inspection

If the cumulative number of lots not accepted in a sequence of consecutive lots on original tightened inspection reaches five, the acceptance procedures of this part of ISO 3951 shall be discontinued.

Inspection under the provisions of this part of ISO 3951 shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service and the responsible authority has agreed that this action is likely to be effective. Tightened inspection shall then be used as if 21.2 has been invoked.

23 Switching between the *s*-method and *σ*-method

23.1 Estimating the process standard deviation

While this part of ISO 3951 is being used, the weighted root mean square of the values of s shall be calculated periodically as estimates of the process standard deviation σ for both the s–method and the σ–method. (See J.2.) The value of σ shall be re-estimated at five-lot intervals, unless the responsible authority specifies another interval. The estimate shall be based on the preceding 10 lots, unless the responsible authority specifies another number of lots.

23.2 State of statistical control

Calculate the upper control limit for each of the 10 lots (or other number of lots specified by the responsible authority) from the expression $c_U\sigma$, where c_U is a factor that depends on the sample size, n, and is given in Table H.1. If none of the sample standard deviations, s_i, exceed the corresponding control limit, then the process may be considered to be in a state of statistical control; otherwise, the process shall be considered to be out of statistical control.

NOTE 1 If the sample sizes from the lots are all equal, then the value of $c_U\sigma$ is common to all the lots.

NOTE 2 If the sample sizes from each lot vary, it is not necessary to calculate $c_U\sigma$ for those lots for which the sample standard deviation, s_i, is less than or equal to σ.

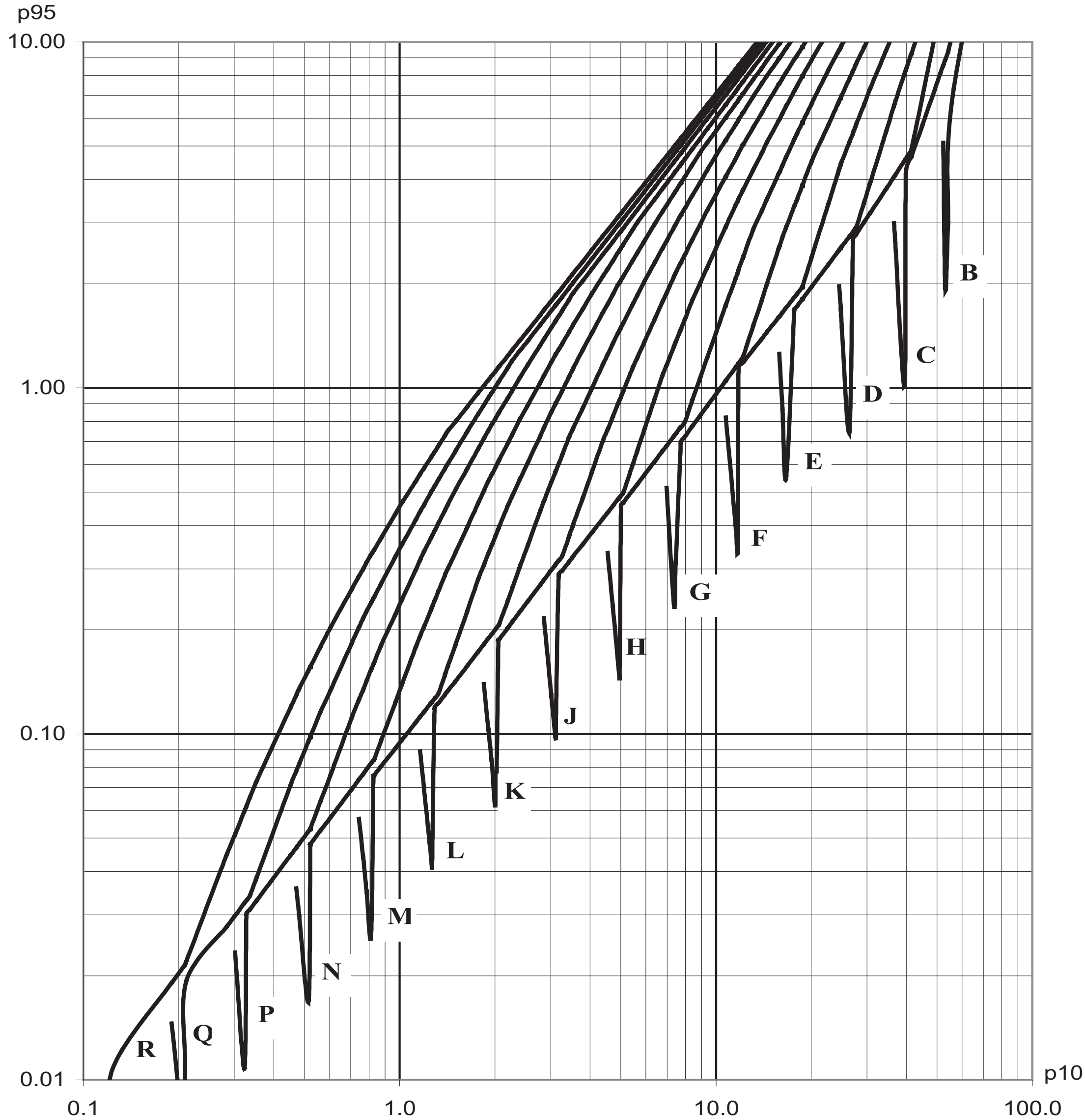
23.3 Switching from the *s*-method to the *σ*-method

If the process is considered to be in a state of statistical control under the s–method, then the σ–method may be instituted using the latest value of σ.

NOTE This switch is made at the discretion of the responsible authority.

23.4 Switching from the *σ*-method to the *s*-method

It is recommended that a control chart for s be kept, even under the σ–method. If s exceeds $c_U\sigma$, the process is considered to be out of statistical control and inspection shall be switched to the s–method.



Key

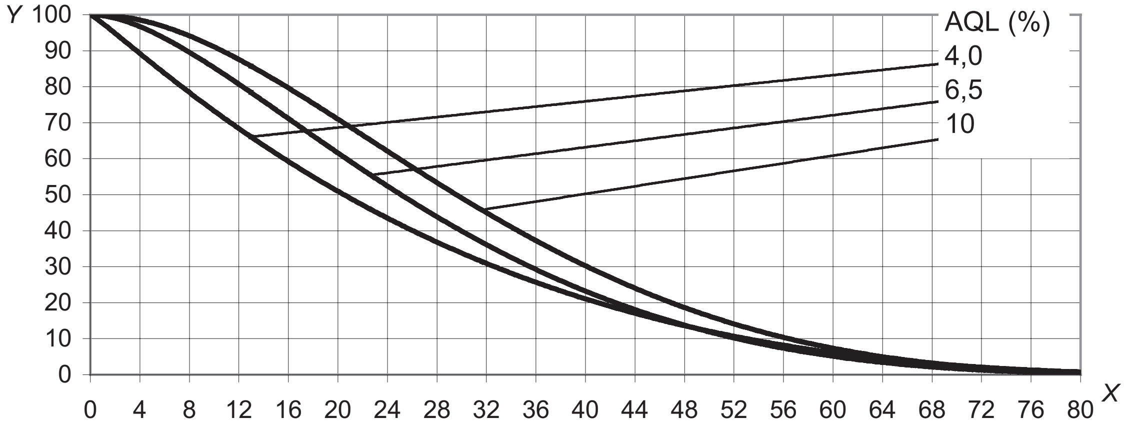
p10 quality level at probability of acceptance 10 % (in percent nonconforming)

p95 quality level at probability of acceptance 95 % (in percent nonconforming)

Figure 4 — Chart A: Sample size code letters of standard single sampling plans for specified qualities at probabilities of acceptance 95 % and 10 %

24 Charts B to R — Operating characteristic curves and tabulated values for single sampling plans, normal inspection: *s*-method

24.1 Operating characteristic curves and tabulated values for sample size code letter B: *s*-method



Key

X process quality (in percent nonconforming)

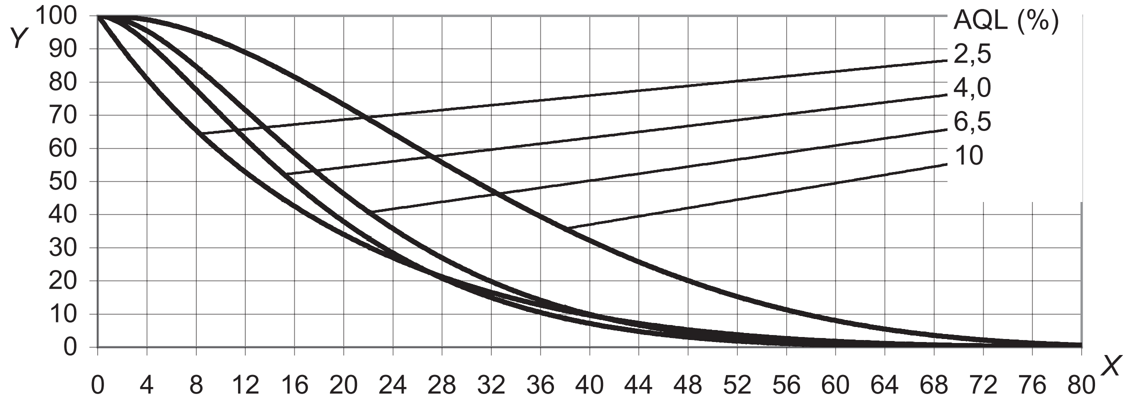
Y percent of lots expected to be accepted (P_a)

Figure 5 — Chart B: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter B					P_a %
	4,0		6,5	10,0		
99,0	0,458	1,34	2,06	3,35	13,65	99,0
95,0	1,94	3,73	5,11	7,37	20,19	95,0
90,0	3,73	5,98	7,80	10,64	24,33	90,0
75,0	9,32	11,85	14,40	18,18	32,15	75,0
50,0	20,49	21,92	25,10	29,57	41,87	50,0
25,0	36,55	35,40	38,75	43,34	52,11	25,0
10,0	53,01	49,17	52,27	56,44	61,22	10,0
5,0	62,60	57,47	60,26	64,00	66,43	5,0
1,0	78,03	71,75	73,82	76,56	75,33	1,0
	6,5		10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter B					
	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter D					

24.2 Operating characteristic curves and tabulated values for sample size code letter C: *s*-method



Key

X process quality (in percent nonconforming)

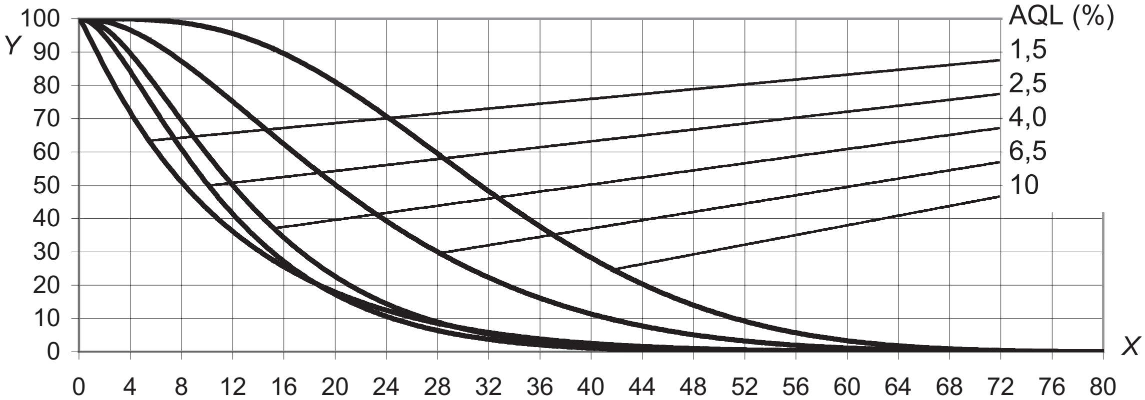
Y percent of lots expected to be accepted (P_a)

Figure 6 — Chart C: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter C						P_a %
	2,5		4,0	6,5	10,0		
99,0	0,224	0,827	1,20	1,87	4,91	15,80	99,0
95,0	1,02	2,25	2,99	4,22	9,38	22,38	95,0
90,0	2,03	3,60	4,61	6,20	12,72	26,44	90,0
75,0	5,45	7,23	8,72	10,98	19,96	33,97	75,0
50,0	13,04	13,79	15,82	18,73	30,33	43,14	50,0
25,0	25,31	23,30	25,69	29,01	42,51	52,70	25,0
10,0	39,48	34,03	36,51	39,88	54,11	61,19	10,0
5,0	48,59	41,10	43,52	46,77	60,93	66,05	5,0
1,0	65,06	54,73	56,84	59,65	72,67	74,43	1,0
	4,0		6,5	10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter C						
	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter E						

24.3 Operating characteristic curves and tabulated values for sample size code letter D: *s*-method



Key

X process quality (in percent nonconforming)

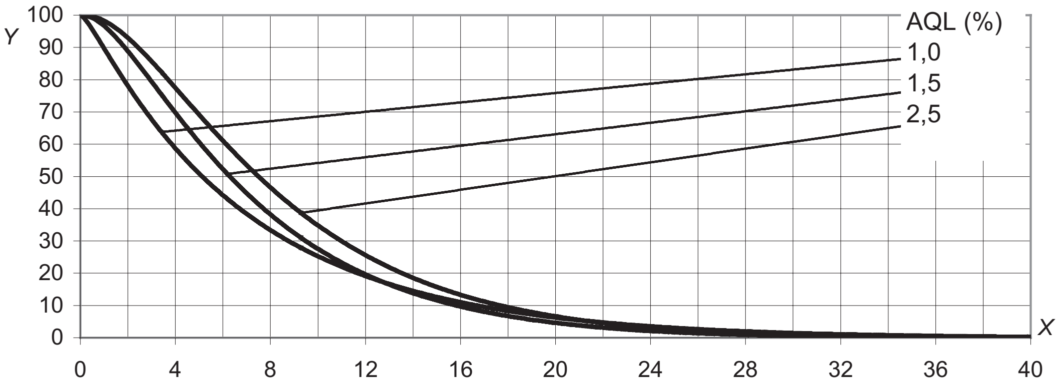
Y percent of lots expected to be accepted (P_a)

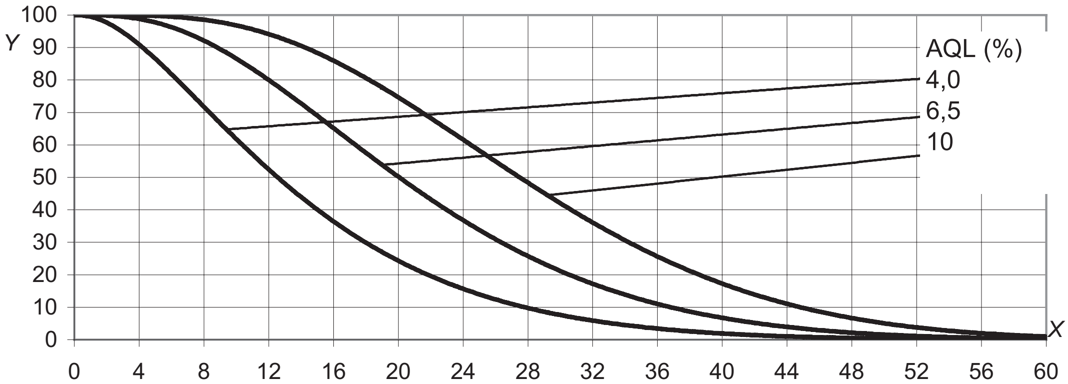
Figure 7 — Chart D: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter D							P_a %
	1,5		2,5	4,0	6,5	10,0		
99,0	0,194	0,488	0,828	1,23	2,23	6,34	14,51	99,0
95,0	0,746	1,35	1,97	2,69	4,84	11,05	20,81	95,0
90,0	1,40	2,18	2,98	3,93	6,99	14,39	24,74	90,0
75,0	3,53	4,49	5,56	6,95	12,05	21,33	32,10	75,0
50,0	8,24	8,89	10,11	12,01	20,07	30,92	41,18	50,0
25,0	16,26	15,70	16,75	19,08	30,50	41,97	50,76	25,0
10,0	26,37	23,94	24,51	27,08	41,37	52,48	59,36	10,0
5,0	33,48	29,70	29,85	32,49	48,20	58,72	64,32	5,0
1,0	47,88	41,62	40,84	43,42	60,87	69,71	72,95	1,0
	2,5		4,0	6,5	10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter D							
	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter F							

24.4 Operating characteristic curves and tabulated values for sample size code letter E: *s*-method





Key

X process quality (in percent nonconforming)

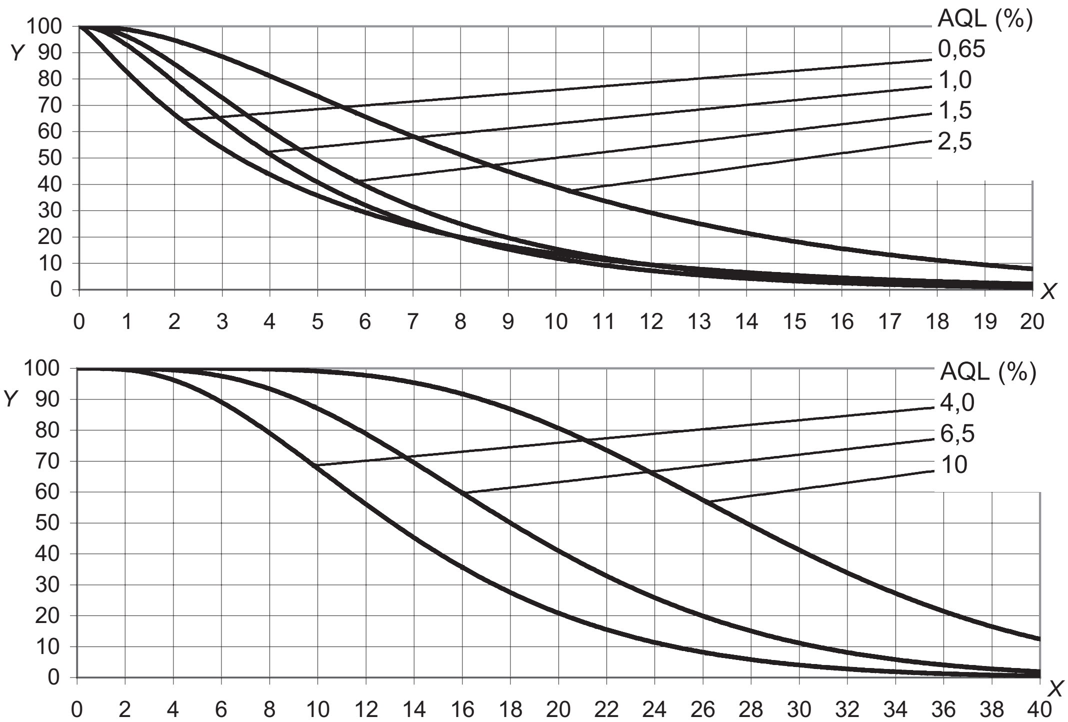
Y percent of lots expected to be accepted (P_a)

Figure 8 — Chart E: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter E								P_a %
	1,0		1,5	2,5	4,0	6,5	10,0		
99,0	0,168	0,313	0,549	0,785	1,36	3,79	7,26	18,19	99,0
95,0	0,552	0,839	1,25	1,68	2,92	6,70	11,44	23,93	95,0
90,0	0,975	1,35	1,87	2,44	4,22	8,83	14,25	27,36	90,0
75,0	2,28	2,77	3,46	4,31	7,36	13,40	19,89	33,57	75,0
50,0	5,13	5,56	6,30	7,51	12,55	20,06	27,50	41,03	50,0
25,0	10,08	10,05	10,58	12,16	19,74	28,31	36,27	48,83	25,0
10,0	16,68	15,80	15,84	17,70	27,80	36,81	44,82	55,90	10,0
5,0	21,59	20,02	19,62	21,62	33,22	42,23	50,06	60,05	5,0
1,0	32,44	29,31	27,87	30,01	44,13	52,64	59,79	67,51	1,0
	1,5		2,5	4,0	6,5	10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter E								
	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter G								

24.5 Operating characteristic curves and tabulated values for sample size code letter F: *s*-method



Key

X process quality (in percent nonconforming)

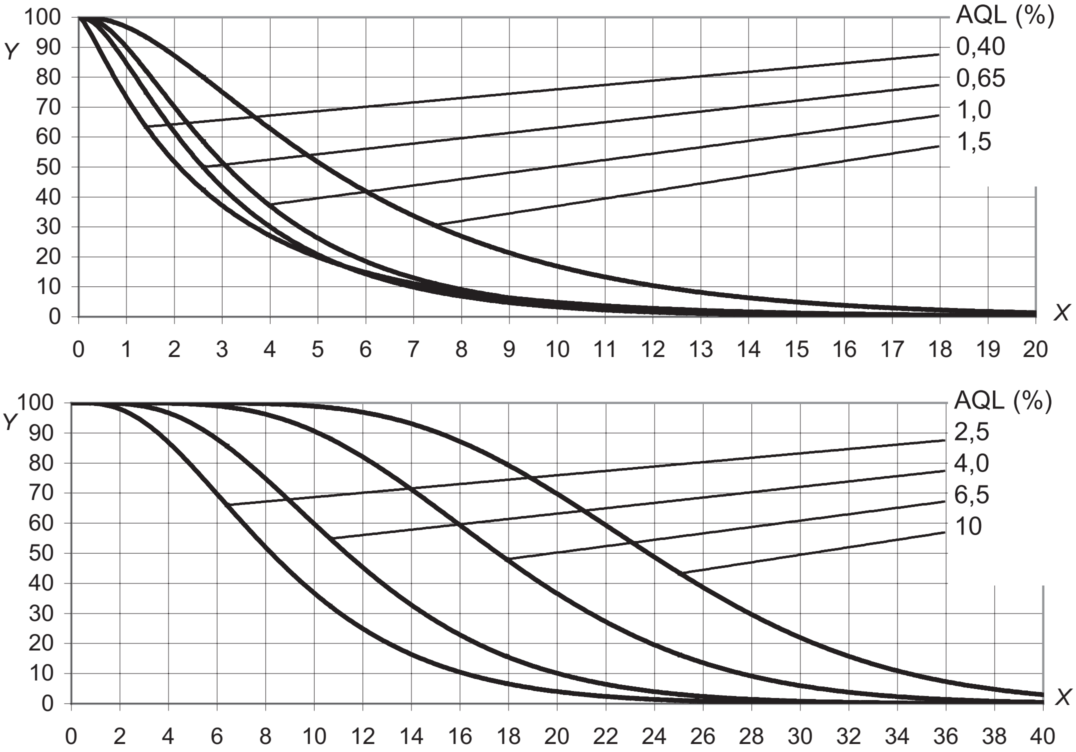
Y percent of lots expected to be accepted (P_a)

Figure 9 — Chart F: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter F									P_a %
	0,65		1,0	1,5	2,5	4,0	6,5	10,0		
99,0	0,102	0,231	0,365	0,552	0,940	2,57	4,67	10,19	13,02	99,0
95,0	0,339	0,584	0,821	1,14	1,95	4,43	7,33	14,24	17,73	95,0
90,0	0,605	0,918	1,22	1,63	2,79	5,79	9,13	16,76	20,63	90,0
75,0	1,45	1,84	2,25	2,84	4,81	8,72	12,84	21,60	26,06	75,0
50,0	3,35	3,63	4,13	4,92	8,21	13,09	18,00	27,80	32,84	50,0
25,0	6,82	6,57	7,04	8,01	13,05	18,71	24,25	34,73	40,23	25,0
10,0	11,70	10,45	10,74	11,79	18,74	24,78	30,70	41,44	47,19	10,0
5,0	15,50	13,39	13,48	14,54	22,71	28,83	34,86	45,58	51,41	5,0
1,0	24,31	20,15	19,69	20,67	31,17	37,07	43,08	53,43	59,23	1,0
	1,0		1,5	2,5	4,0	6,5	10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter F									
	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter H									

24.6 Operating characteristic curves and tabulated values for sample size code letter G: *s*-method



Key

X process quality (in percent nonconforming)

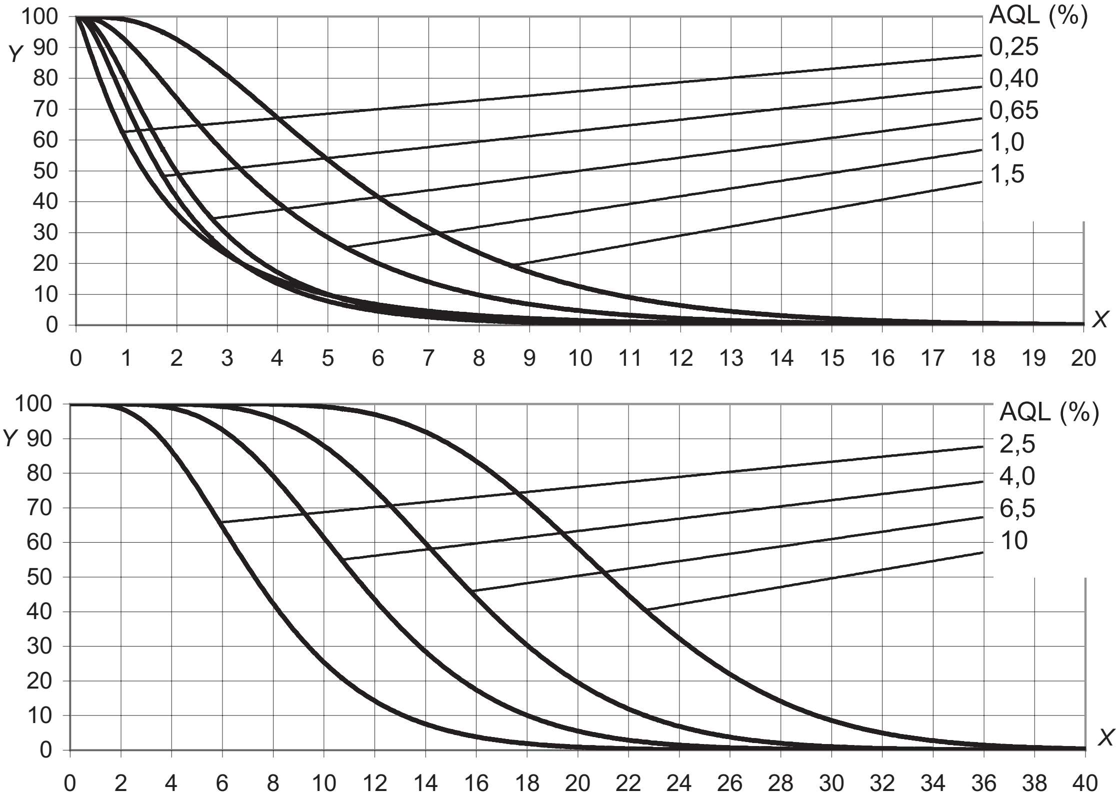
Y percent of lots expected to be accepted (P_a)

Figure 10 — Chart G: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter G											P_a %
	0,40		0,65	1,0	1,5	2,5	4,0	6,5		10,0		
99,00	0,077 2	0,144	0,231	0,335	0,601	1,58	2,88	6,02	7,85	9,85	11,96	99,0
95,0	0,236	0,362	0,514	0,697	1,23	2,73	4,51	8,54	10,76	13,12	15,56	95,0
90,0	0,406	0,568	0,762	1,00	1,75	3,57	5,63	10,16	12,59	15,13	17,74	90,0
75,0	0,932	1,14	1,41	1,76	3,00	5,41	7,97	13,34	16,09	18,93	21,79	75,0
50,0	2,10	2,28	2,60	3,10	5,16	8,24	11,32	17,57	20,65	23,75	26,84	50,0
25,0	4,25	4,21	4,49	5,14	8,32	11,99	15,53	22,53	25,86	29,16	32,40	25,0
10,0	7,37	6,85	6,97	7,73	12,17	16,21	20,04	27,56	31,05	34,46	37,76	10,0
5,0	9,86	8,90	8,86	9,67	14,96	19,12	23,06	30,81	34,34	37,78	41,09	5,0
1,0	15,94	13,82	13,30	14,13	21,13	25,29	29,30	37,26	40,81	44,22	47,47	1,0
	0,65		1,0	1,5	2,5	4,0	6,5		10,0			
	Acceptance quality limit (tightened inspection) in percent — sample size code letter G											
	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5		10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter J											

24.7 Operating characteristic curves and tabulated values for sample size code letter H: *s*-method



Key

X process quality (in percent nonconforming)

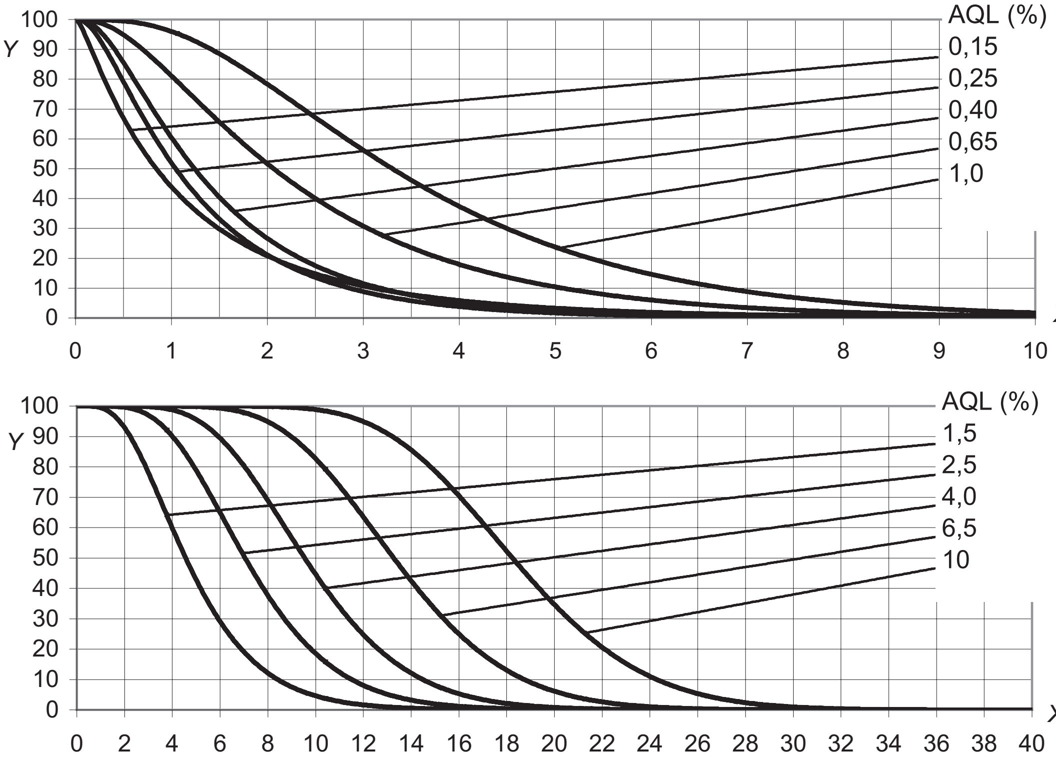
Y percent of lots expected to be accepted (P_a)

Figure 11 — Chart H: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter H												P_a %
	0,25		0,40	0,65	1,0	1,5	2,5	4,0		6,5		10,0	
99,0	0,047 8	0,096 6	0,153	0,225	0,398	1,01	1,85	3,85	5,03	6,22	7,49	10,21	99,0
95,0	0,146	0,237	0,334	0,457	0,797	1,73	2,88	5,43	6,85	8,29	9,77	12,90	95,0
90,0	0,253	0,368	0,492	0,650	1,13	2,26	3,60	6,46	8,01	9,57	11,18	14,51	90,0
75,0	0,586	0,734	0,901	1,13	1,92	3,45	5,09	8,49	10,25	12,03	13,82	17,49	75,0
50,0	1,35	1,47	1,67	1,98	3,31	5,29	7,27	11,24	13,22	15,22	17,21	21,21	50,0
25,0	2,79	2,72	2,90	3,30	5,38	7,80	10,06	14,55	16,71	18,91	21,07	25,34	25,0
10,0	4,96	4,47	4,54	5,01	7,96	10,71	13,14	18,01	20,28	22,64	24,92	29,37	10,0
5,0	6,75	5,87	5,82	6,30	9,87	12,76	15,24	20,30	22,61	25,04	27,38	31,91	5,0
1,0	11,27	9,32	8,90	9,36	14,25	17,25	19,72	24,99	27,34	29,87	32,28	36,89	1,0
	0,40		0,65	1,0	1,5	2,5	4,0	6,5			10,0		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter H												
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0		6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent — sample size code letter K												

24.8 Operating characteristic curves and tabulated values for sample size code letter J: *s*-method



Key

X process quality (in percent nonconforming)

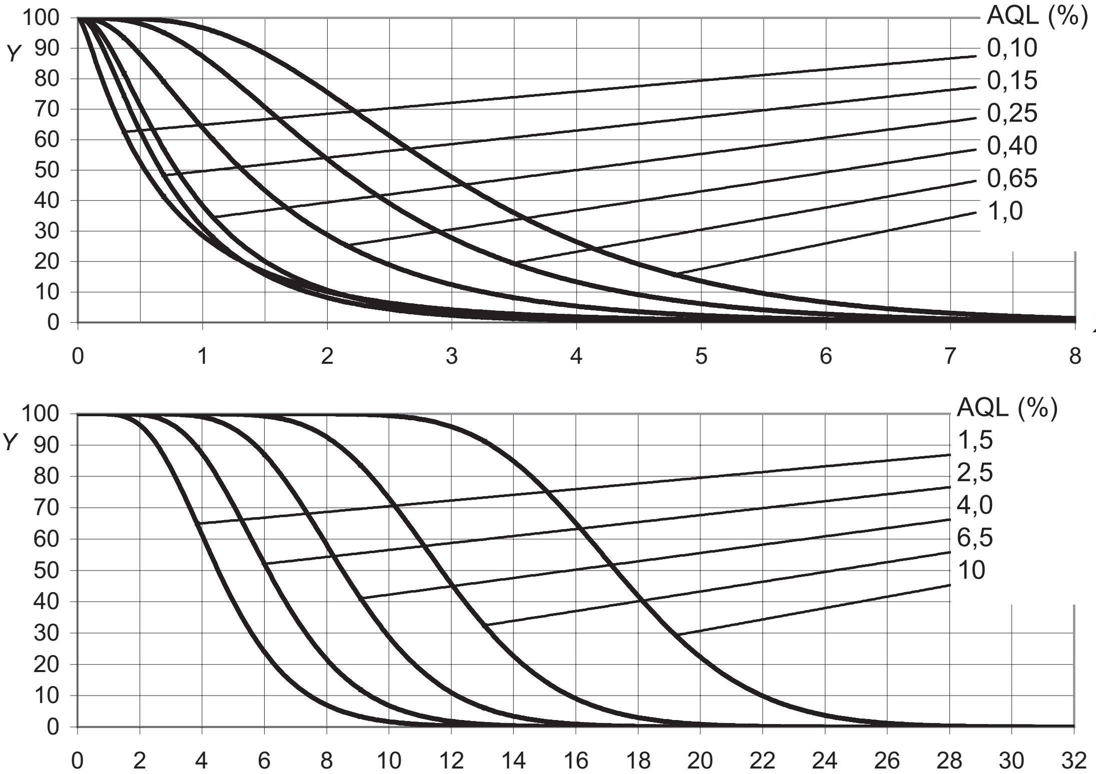
Y percent of lots expected to be accepted (P_a)

Figure 12 — Chart J: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter J														P_a %
	0,15		0,25	0,40	0,65	1,0	1,5	2,5		4,0		6,5		10,0	
99,0,0	0,033 4	0,0625	0,102	0,144	0,251	0,645	1,15	2,39	3,11	3,84	4,62	6,30	8,06	9,86	99,0
95,0	0,097 1	0,150	0,216	0,288	0,498	1,09	1,79	3,37	4,24	5,12	6,04	7,96	9,95	11,97	95,0
90,0	0,164	0,232	0,315	0,408	0,701	1,42	2,23	4,01	4,97	5,93	6,92	8,97	11,08	13,21	90,0
75,0	0,372	0,459	0,569	0,707	1,20	2,16	3,17	5,29	6,39	7,48	8,60	10,86	13,16	15,46	75,0
50,0	0,841	0,915	1,04	1,24	2,07	3,31	4,56	7,05	8,30	9,54	10,79	13,27	15,75	18,24	50,0
25,0	1,74	1,71	1,81	2,08	3,39	4,90	6,37	9,20	10,59	11,97	13,34	16,00	18,65	21,30	25,0
10,0	3,11	2,84	2,86	3,18	5,09	6,78	8,41	11,51	12,99	14,48	15,94	18,74	21,52	24,29	10,0
5,0	4,27	3,76	3,68	4,03	6,37	8,13	9,84	13,06	14,59	16,14	17,64	20,50	23,34	26,17	5,0
1,0	7,27	6,08	5,70	6,09	9,38	11,16	12,93	16,32	17,91	19,53	21,10	24,04	26,96	29,89	1,0
	0,25		0,40	0,65	1,0	1,5	2,5	4,0			6,5		10,0		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter J														
	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5		4,0	6,5			
	Acceptance quality limit (reduced inspection) in percent — sample size code letter L														

24.9 Operating characteristic curves and tabulated values for sample size code letter K: *s*-method



Key

X process quality (in percent nonconforming)

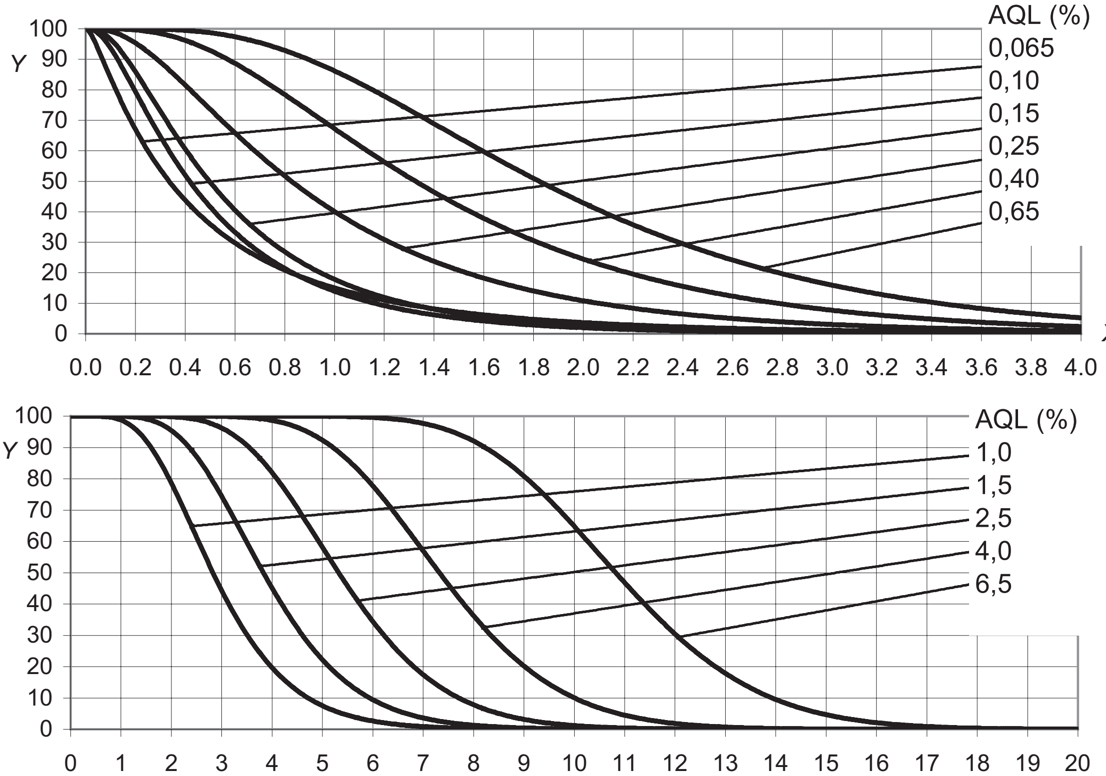
Y percent of lots expected to be accepted (P_a)

Figure 13 — Chart K: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter K																P_a %
	0,10		0,15	0,25	0,40	0,65	1,0	1,5		2,5		4,0		6,5		10,0	
99,0	0,022 7	0,042 2	0,066 4	0,094 4	0,169	0,420	0,741	1,53	1,98	2,45	2,94	3,99	5,09	6,22	8,64	10,49	99,0
95,0	0,064 0	0,098 8	0,139	0,187	0,328	0,703	1,14	2,15	2,70	3,27	3,84	5,06	6,30	7,57	10,23	12,25	95,0
90,0	0,107	0,151	0,202	0,263	0,457	0,912	1,43	2,56	3,16	3,78	4,40	5,71	7,03	8,37	11,16	13,27	90,0
75,0	0,239	0,295	0,364	0,453	0,772	1,38	2,02	3,38	4,08	4,78	5,48	6,93	8,38	9,84	12,83	15,09	75,0
50,0	0,539	0,585	0,667	0,794	1,32	2,12	2,92	4,52	5,32	6,11	6,91	8,51	10,09	11,69	14,88	17,29	50,0
25,0	1,12	1,09	1,16	1,33	2,17	3,15	4,10	5,93	6,83	7,71	8,59	10,33	12,04	13,76	17,14	19,68	25,0
10,0	2,01	1,82	1,85	2,05	3,27	4,39	5,45	7,46	8,45	9,39	10,33	12,18	13,98	15,81	19,33	21,98	10,0
5,0	2,78	2,43	2,39	2,61	4,10	5,29	6,41	8,50	9,53	10,51	11,49	13,39	15,24	17,12	20,72	23,43	5,0
1,0	4,83	3,98	3,75	3,99	6,11	7,35	8,53	10,73	11,82	12,84	13,88	15,87	17,79	19,76	23,47	26,27	1,0
	0,15		0,25	0,40	0,65	1,0	1,5	2,5			4,0		6,5		10,0		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter K																
	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5		2,5	4,0					
	Acceptance quality limit (reduced inspection) in percent — sample size code letter M																

24.10 Operating characteristic curves and tabulated values for sample size code letter L: *s*-method



Key

X process quality (in percent nonconforming)

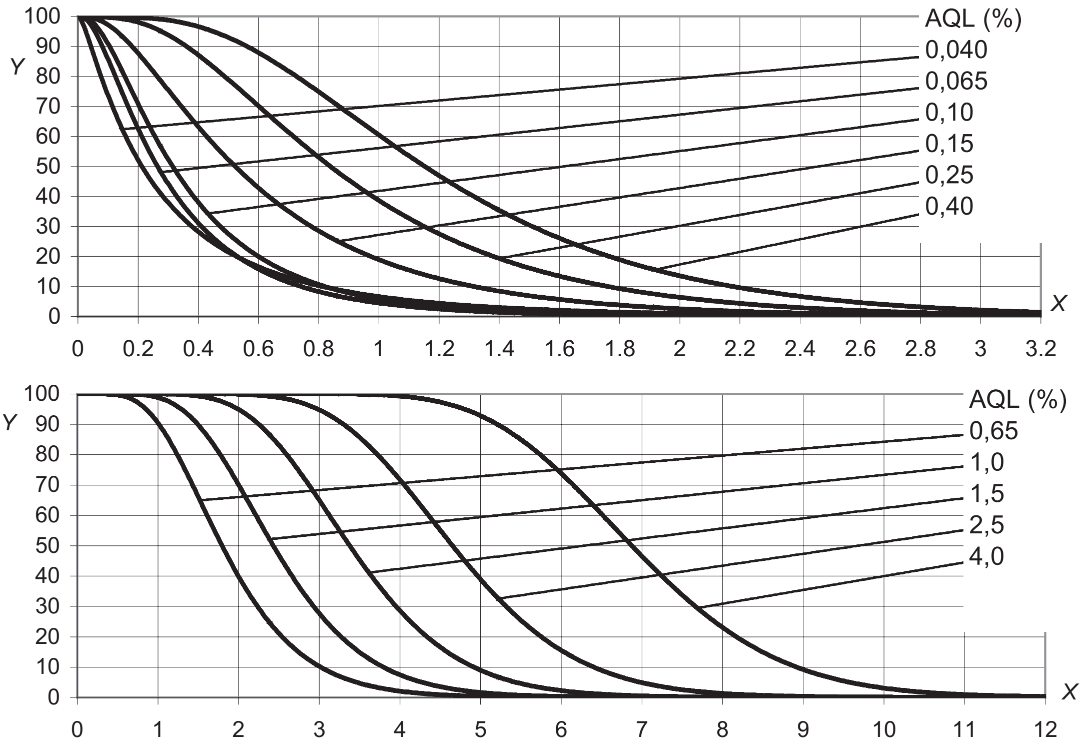
Y percent of lots expected to be accepted (P_a)

Figure 14 — Chart L: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter L																P_a %
	0,065		0,10	0,15	0,25	0,40	0,65	1,0		1,5		2,5		4,0		6,5	
99,0	0,014 9	0,026 6	0,043 2	0,061 0	0,108	0,264	0,470	0,959	1,24	1,53	1,84	2,48	3,15	3,86	5,34	6,47	99,0
95,0	0,041 0	0,061 8	0,089 0	0,119	0,207	0,440	0,720	1,34	1,68	2,03	2,40	3,14	3,91	4,70	6,33	7,58	95,0
90,0	0,068 0	0,094 4	0,128	0,166	0,287	0,570	0,894	1,60	1,97	2,35	2,75	3,55	4,37	5,21	6,92	8,23	90,0
75,0	0,150	0,184	0,229	0,285	0,483	0,862	1,27	2,11	2,54	2,98	3,42	4,32	5,22	6,14	7,99	9,39	75,0
50,0	0,336	0,367	0,418	0,497	0,827	1,33	1,82	2,83	3,32	3,83	4,32	5,32	6,31	7,32	9,31	10,82	50,0
25,0	0,697	0,690	0,729	0,835	1,36	1,98	2,57	3,73	4,28	4,85	5,39	6,50	7,57	8,66	10,78	12,39	25,0
10,0	1,27	1,16	1,16	1,29	2,06	2,78	3,43	4,72	5,31	5,94	6,52	7,71	8,85	10,00	12,24	13,93	10,0
5,0	1,76	1,56	1,51	1,65	2,60	3,37	4,04	5,40	6,02	6,67	7,27	8,50	9,68	10,87	13,16	14,91	5,0
1,0	3,11	2,60	2,39	2,54	3,92	4,72	5,42	6,87	7,52	8,22	8,84	10,16	11,39	12,64	15,03	16,85	1,0
	0,10		0,15	0,25	0,40	0,65	1,0	1,5			2,5		4,0		6,5		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter L																
	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0		1,5	2,5					
	Acceptance quality limit (reduced inspection) in percent — sample size code letter N																

24.11 Operating characteristic curves and tabulated values for sample size code letter M: *s*-method



Key

X process quality (in percent nonconforming)

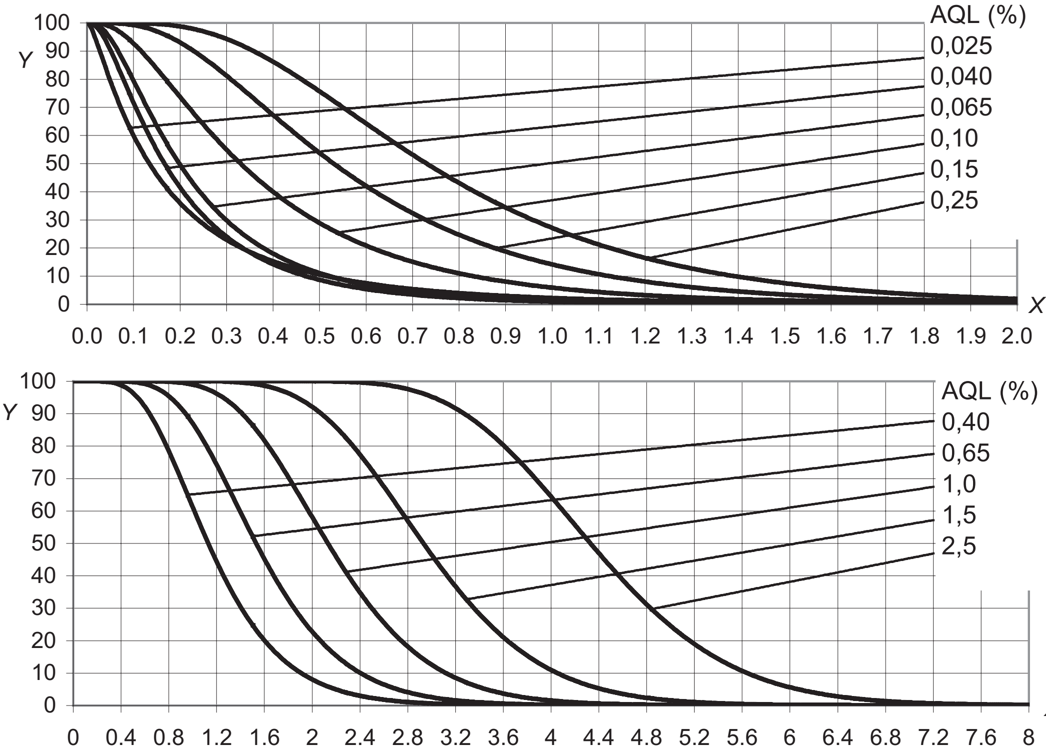
Y percent of lots expected to be accepted (P_a)

Figure 15 — Chart M: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter M																P_a %
	0,04		0,065	0,10	0,15	0,25	0,40	0,65		1,0		1,5		2,5		4,0	
99,0	0,009 7	0,017 2	0,027 8	0,039 1	0,068 5	0,170	0,299	0,610	0,787	0,973	1,17	1,58	2,00	2,44	3,36	4,09	99,0
95,0	0,026 3	0,039 5	0,056 8	0,075 5	0,131	0,282	0,457	0,85	1,07	1,29	1,52	1,99	2,48	2,98	4,00	4,79	95,0
90,0	0,043 4	0,060 1	0,081 6	0,106	0,181	0,364	0,567	1,01	1,25	1,49	1,74	2,25	2,77	3,30	4,38	5,20	90,0
75,0	0,095 2	0,117	0,145	0,180	0,305	0,548	0,803	1,34	1,61	1,89	2,17	2,74	3,31	3,89	5,06	5,95	75,0
50,0	0,213	0,233	0,265	0,315	0,524	0,842	1,16	1,80	2,11	2,43	2,75	3,38	4,02	4,65	5,92	6,87	50,0
25,0	0,444	0,439	0,465	0,532	0,868	1,26	1,64	2,37	2,73	3,09	3,44	4,13	4,84	5,52	6,89	7,89	25,0
10,0	0,813	0,746	0,743	0,826	1,33	1,77	2,19	3,02	3,40	3,79	4,17	4,91	5,68	6,39	7,85	8,91	10,0
5,0	1,14	1,00	0,97	1,06	1,68	2,16	2,60	3,46	3,86	4,27	4,66	5,43	6,23	6,97	8,47	9,55	5,0
1,0	2,04	1,69	1,55	1,65	2,57	3,05	3,51	4,44	4,85	5,29	5,70	6,51	7,37	8,14	9,73	10,86	1,0
	0,065		0,10	0,15	0,25	0,40	0,65	1,0			1,5		2,5		4,0		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter M																
	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65		1,0	1,5					
	Acceptance quality limit (reduced inspection) in percent — sample size code letter P																

24.12 Operating characteristic curves and tabulated values for sample size code letter N: *s*-method



Key

X process quality (in percent nonconforming)

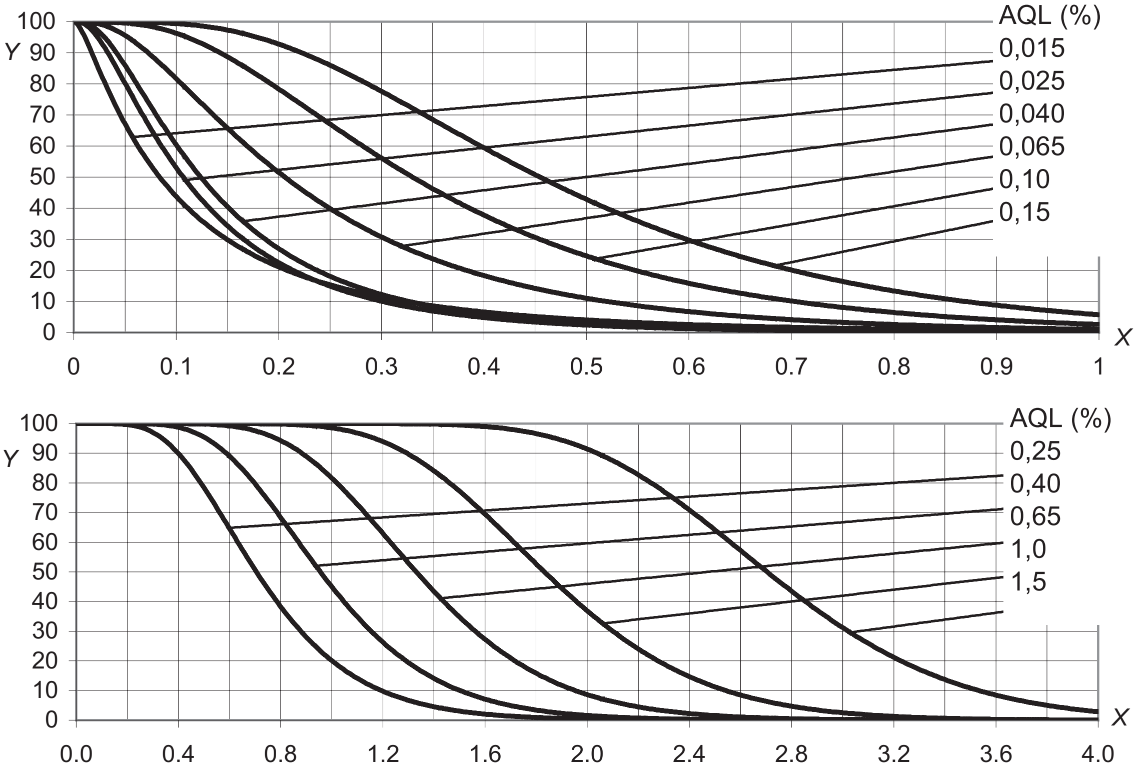
Y percent of lots expected to be accepted (P_a)

Figure 16 — Chart N: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter N																P_a %
	0,025		0,04	0,065	0,10	0,15	0,25	0,40		0,65		1,0		1,5		2,5	
99,0	0,006 4	0,011 1	0,017 6	0,025 0	0,0445	0,109	0,190	0,387	0,497	0,614	0,735	0,991	1,26	1,54	2,12	2,57	99,0
95,0	0,016 9	0,025 1	0,035 8	0,048 0	0,083 7	0,178	0,289	0,538	0,673	0,813	0,956	1,25	1,56	1,87	2,51	3,01	95,0
90,0	0,027 6	0,038 0	0,051 3	0,066 8	0,115	0,230	0,358	0,639	0,787	0,940	1,10	1,41	1,74	2,07	2,75	3,27	90,0
75,0	0,060 2	0,073 7	0,091 2	0,114	0,193	0,345	0,506	0,842	1,01	1,19	1,37	1,72	2,08	2,45	3,19	3,74	75,0
50,0	0,134	0,146	0,167	0,198	0,330	0,531	0,730	1,13	1,33	1,53	1,73	2,13	2,53	2,93	3,73	4,33	50,0
25,0	0,280	0,277	0,293	0,335	0,547	0,796	1,03	1,50	1,72	1,95	2,17	2,61	3,05	3,48	4,35	4,98	25,0
10,0	0,515	0,473	0,471	0,521	0,836	1,12	1,39	1,91	2,15	2,40	2,64	3,12	3,58	4,05	4,97	5,64	10,0
5,0	0,725	0,640	0,618	0,671	1,06	1,37	1,65	2,20	2,45	2,71	2,96	3,45	3,94	4,42	5,37	6,06	5,0
1,0	1,32	1,09	1,00	1,05	1,63	1,95	2,25	2,83	3,10	3,38	3,64	4,16	4,67	5,18	6,19	6,91	1,0
	0,04		0,065	0,10	0,15	0,25	0,40	0,65			1,0		1,5		2,5		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter N																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40		0,65	1,0					
	Acceptance quality limit (reduced inspection) in percent — sample size code letter Q																

24.13 Operating characteristic curves and tabulated values for sample size code letter P: *s*-method



Key

X process quality (in percent nonconforming)

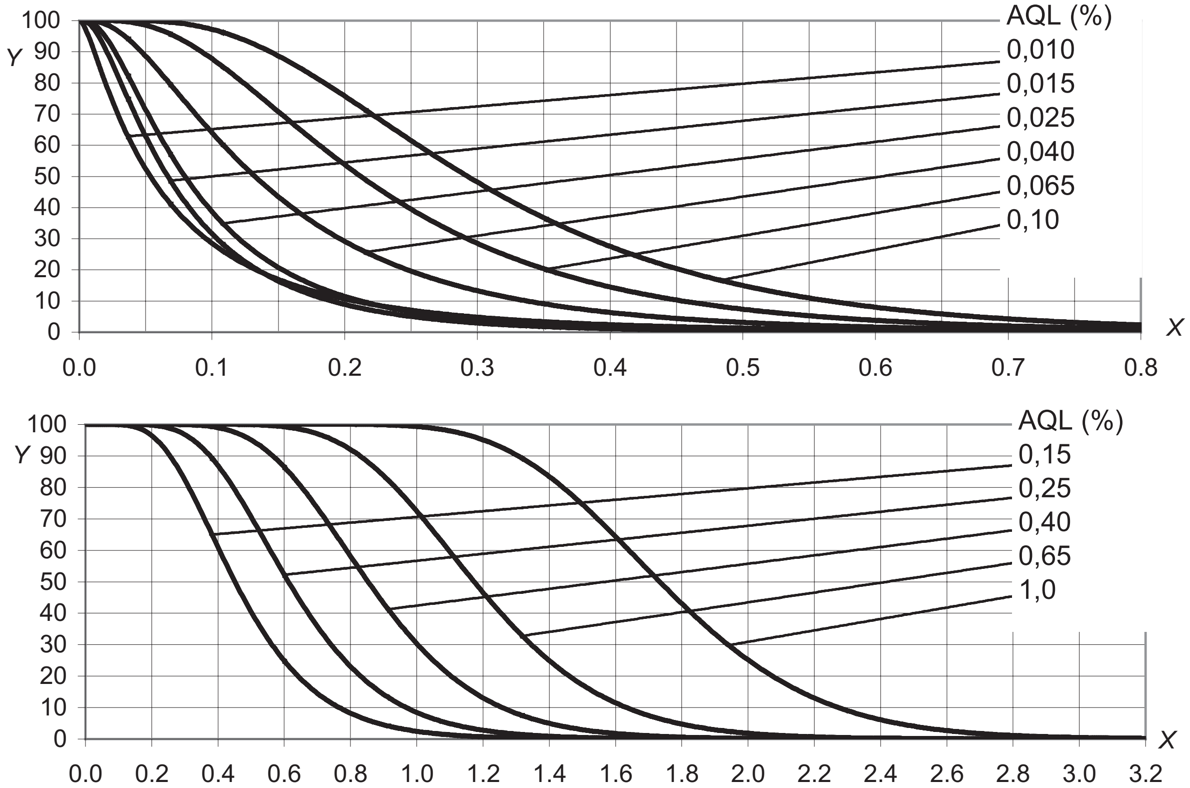
Y percent of lots expected to be accepted (P_a)

Figure 17 — Chart P: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter P																P_a %
	0,015		0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0		1,5	
99,0	,0041	,0070	,0113	,0158	,0286	,0682	0,119	0,243	0,312	0,385	0,461	0,620	0,787	0,960	1,32	1,60	99,0
95,0	,0108	,0158	,0227	,0301	,0531	0,112	0,180	0,337	0,421	0,509	0,598	0,783	0,973	1,17	1,57	1,88	95,0
90,0	,0175	,0239	,0323	,0419	,0728	0,143	0,223	0,399	0,492	0,588	0,685	0,883	1,09	1,29	1,72	2,04	90,0
75,0	,0378	,0461	,0573	,0710	0,121	0,215	0,316	0,526	0,634	0,743	0,854	1,08	1,30	1,53	1,99	2,34	75,0
50,0	,0838	,0914	,104	,124	0,206	0,331	0,456	0,705	0,831	0,956	1,08	1,33	1,58	1,83	2,33	2,70	50,0
25,0	,175	,173	,183	,210	0,341	0,498	0,647	0,936	1,08	1,22	1,36	1,63	1,91	2,18	2,72	3,12	25,0
10,0	,323	,297	,296	,328	0,522	0,705	0,873	1,19	1,35	1,50	1,66	1,95	2,25	2,54	3,11	3,53	10,0
5,0	,456	,404	,389	,423	0,665	0,862	1,04	1,38	1,54	1,70	1,86	2,17	2,47	2,78	3,36	3,80	5,0
1,0	,836	,694	,632	,668	1,03	1,24	1,42	1,78	1,95	2,12	2,30	2,62	2,94	3,27	3,88	4,35	1,0
	0,025		0,04	0,065	0,10	0,15	0,25	0,40			0,65		1,0		1,5		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter P																
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40	0,65					
	Acceptance quality limit (reduced inspection) in percent — sample size code letter R																

24.14 Operating characteristic curves and tabulated values for sample size code letter Q: *s*-method



Key

X process quality (in percent nonconforming)

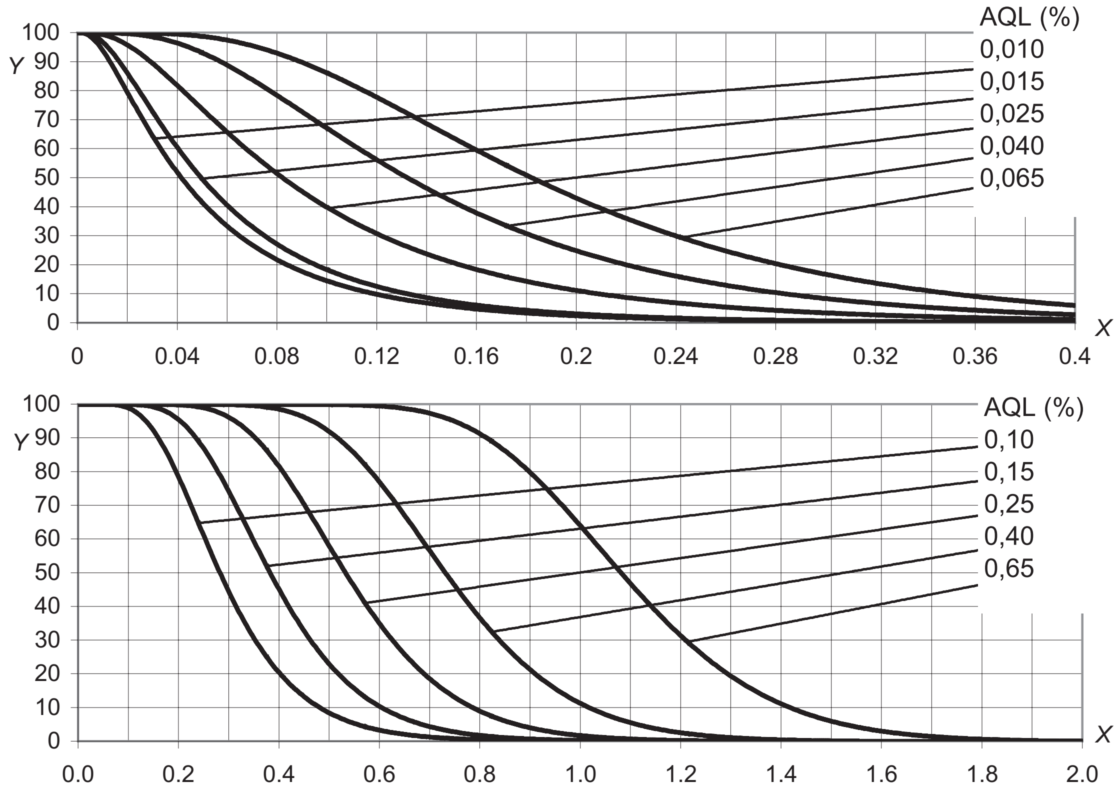
Y percent of lots expected to be accepted (P_a)

Figure 18 — Chart Q: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter Q													P_a %
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0	
99,0	0,0027	0,0073	0,0102	0,0184	0.0440	0,0767	0,156	0,246	0,295	0,396	0,504	0,614	0,845	1,02	99,0
95,0	0,0070	0,0146	0,0193	0,0341	0.0715	0,116	0,216	0,325	0,383	0,500	0,622	0,747	1,00	1,20	95,0
90,0	0,0113	0,0207	0,0269	0,0467	0.0919	0,143	0,256	0,376	0,438	0,565	0,695	0,827	1,10	1,30	90,0
75,0	0,0242	0,0366	0,0455	0,0775	0.138	0,202	0,336	0,476	0,546	0,689	0,833	0,978	1,27	1,49	75,0
50,0	0,0536	0,0667	0,0795	0,132	0.212	0,292	0,451	0,613	0,693	0,853	1,01	1,17	1,49	1,73	50,0
25,0	0,112	0,117	0,135	0,219	0.319	0,415	0,599	0,782	0,872	1,05	1,22	1,40	1,74	2,00	25,0
10,0	0,208	0,190	0,212	0,336	0.453	0,562	0,766	0,968	1,06	1,26	1,44	1,63	2,00	2,27	10,0
5,0	0,294	0,250	0,274	0,430	0.555	0,670	0,883	1,10	1,20	1,40	1,59	1,79	2,16	2,44	5,0
1,0	0,544	0,410	0,435	0,669	0.799	0,920	1,14	1,37	1,48	1,70	1,90	2,11	2,50	2,80	1,0
	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter Q													

24.15 Operating characteristic curves and tabulated values for sample size code letter R: *s*-method



Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

Figure 19 — Chart R: Operating characteristic curves for single sampling plans, normal inspection

Tabulated values for operating characteristic curves for single sampling plans

P_a %	Acceptance quality limit (normal inspection) in percent — sample size code letter R														P_a %
		0,01	0,015	0,025	0,04	0,065	0,10	0,15		0,25		0,40		0,65	
99,0	0,007 0	0,004 6	0,006 5	0,011 5	0,027 7	0,048 2	0,097 6	0,312	0,155	0,461	0,249	0,315	0,384	0,528	99,0
95,0	0,015 8	0,009 2	0,012 2	0,021 2	0,044 9	0,072 6	0,135	0,421	0,204	0,598	0,313	0,389	0,467	0,627	95,0
90,0	0,023 9	0,013 0	0,016 8	0,029 0	0,057 6	0,089 7	0,160	0,492	0,235	0,685	0,353	0,434	0,517	0,686	90,0
75,0	0,046 1	0,022 9	0,028 5	0,048 2	0,086 2	0,126	0,210	0,634	0,297	0,854	0,430	0,521	0,611	0,795	75,0
50,0	0,091 4	0,041 6	0,049 6	0,082 4	0,132	0,182	0,282	0,831	0,382	1,08	0,533	0,633	0,732	0,932	50,0
25,0	0,173	0,073	0,084	0,137	0,200	0,259	0,376	1,08	0,489	1,36	0,656	0,766	0,873	1,09	25,0
10,0	0,297	0,119	0,132	0,212	0,284	0,352	0,481	1,35	0,605	1,66	0,786	0,906	1,02	1,25	10,0
5,0	0,404	0,157	0,172	0,272	0,349	0,420	0,556	1,54	0,685	1,86	0,874	0,999	1,12	1,35	5,0
1,0	0,694	0,259	0,274	0,426	0,505	0,579	0,723	1,95	0,861	2,30	1,06	1,20	1,32	1,57	1,0
	0,010	0,015	0,025	0,04	0,065	0,10	0,15		0,25		0,40		0,65		
	Acceptance quality limit (tightened inspection) in percent — sample size code letter R														

25 Charts s-D to s-R — Acceptance curves for combined control of double specification limits: *s*-method

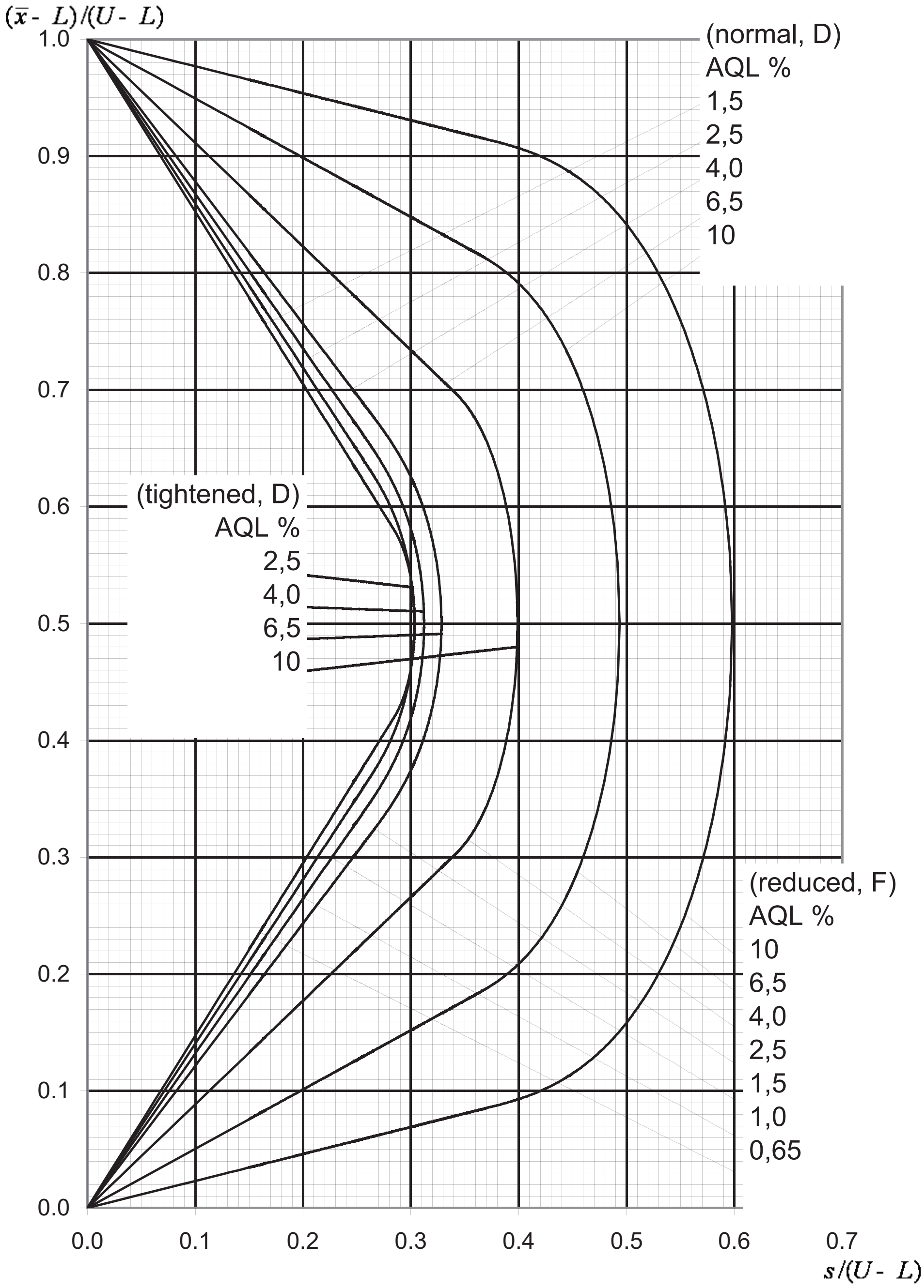


Figure 20 — Chart s-D: Acceptance curves for combined control of double specification limits for sample size code letter D under normal and tightened inspection and for sample size code letter F under reduced inspection

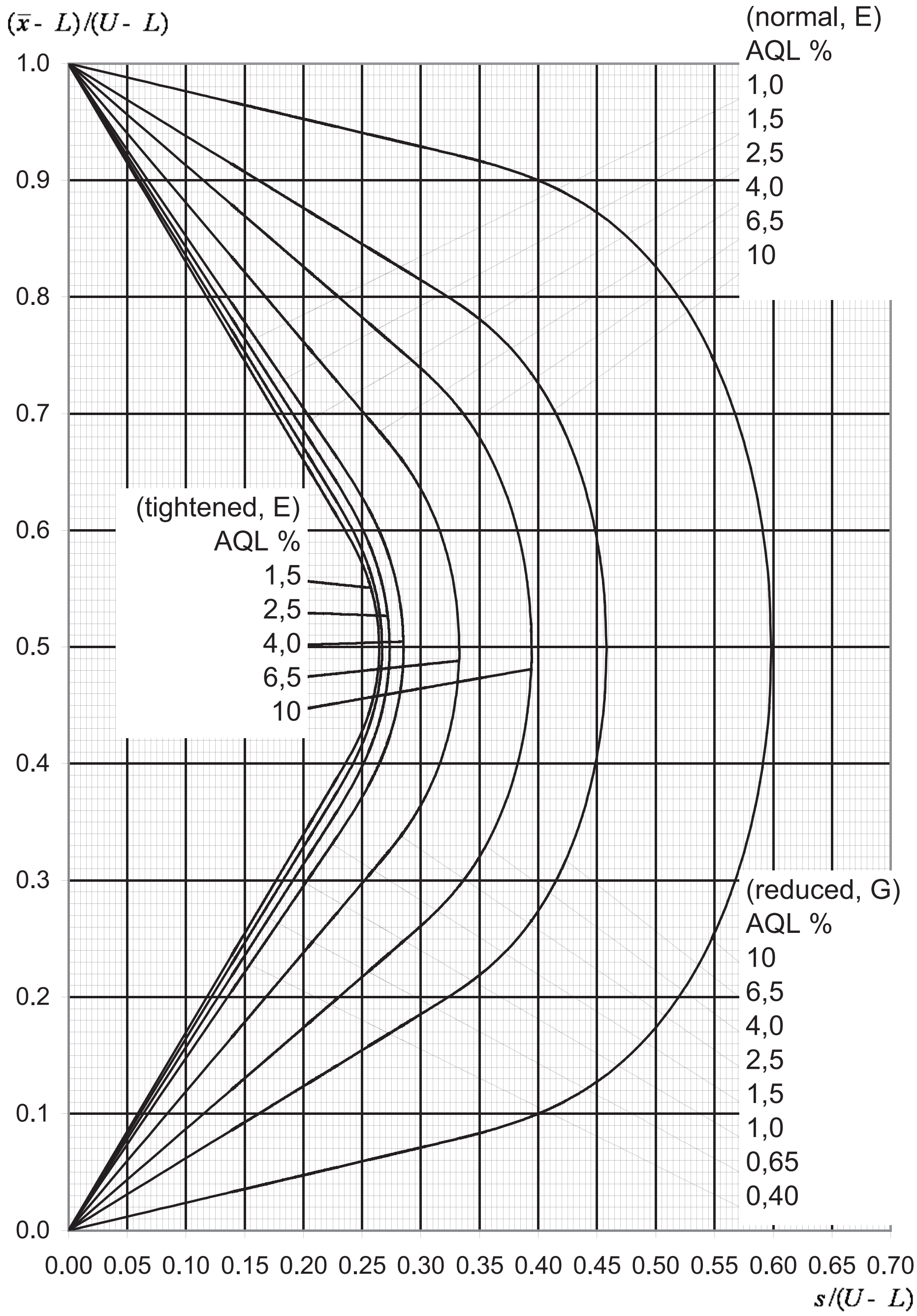


Figure 21 — Chart s-E: Acceptance curves for combined control of double specification limits for sample size code letter E under normal and tightened inspection and for sample size code letter G under reduced inspection

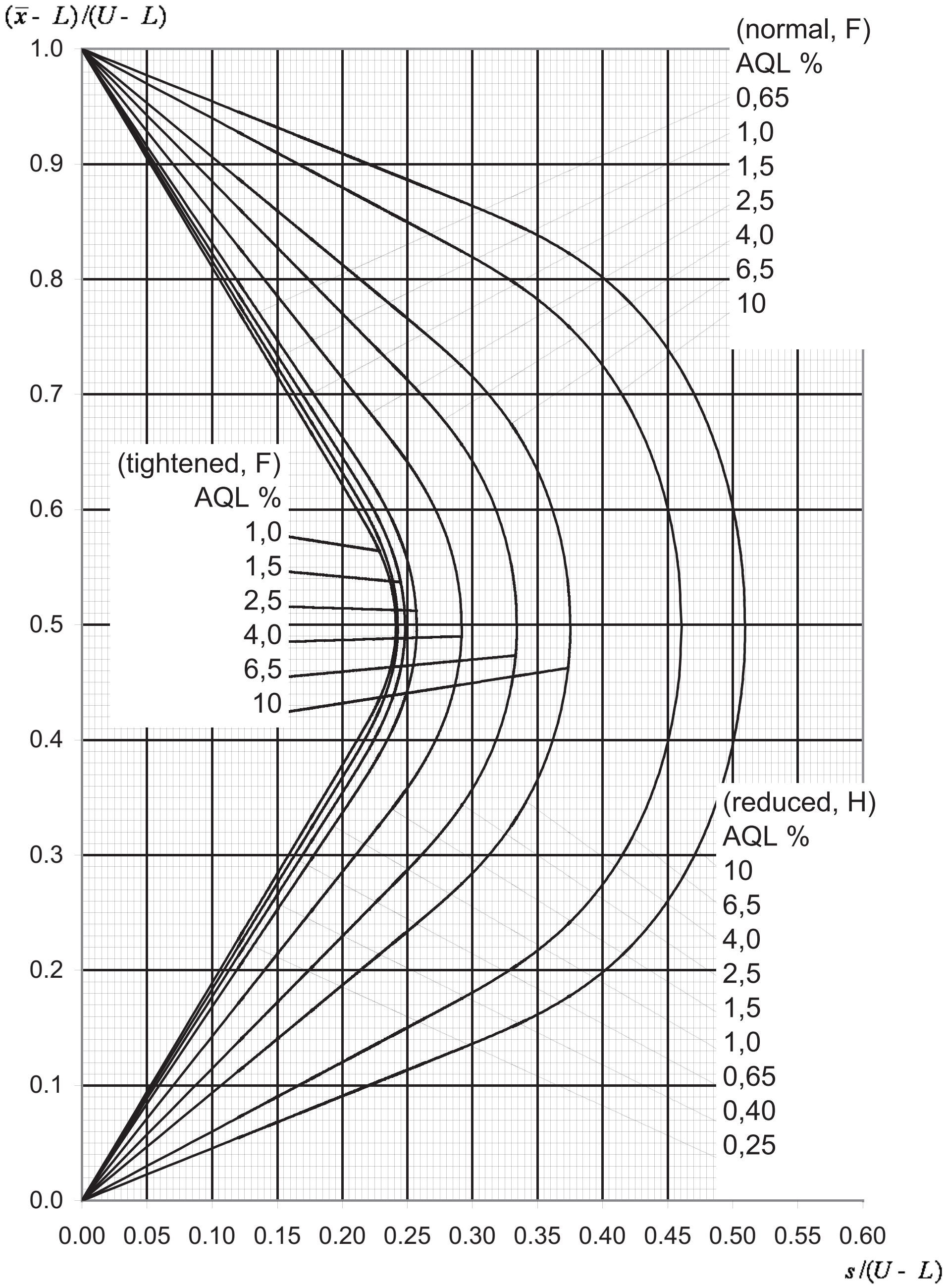


Figure 22 — Chart s-F: Acceptance curves for combined control of double specification limits for sample size code letter F under normal and tightened inspection and for sample size code letter H under reduced inspection

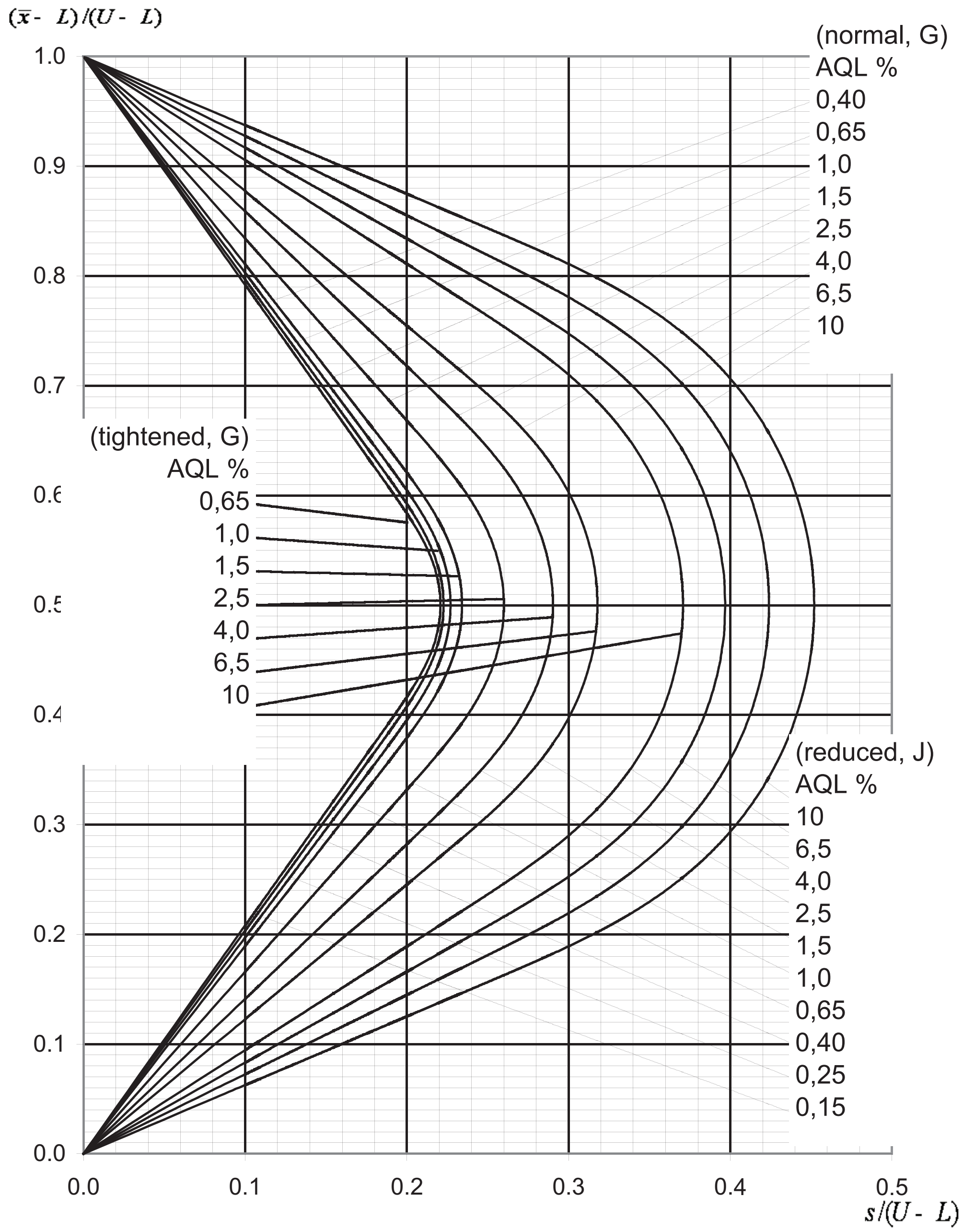


Figure 23 — Chart s-G: Acceptance curves for combined control of double specification limits for sample size code letter G under normal and tightened inspection and for sample size code letter J under reduced inspection

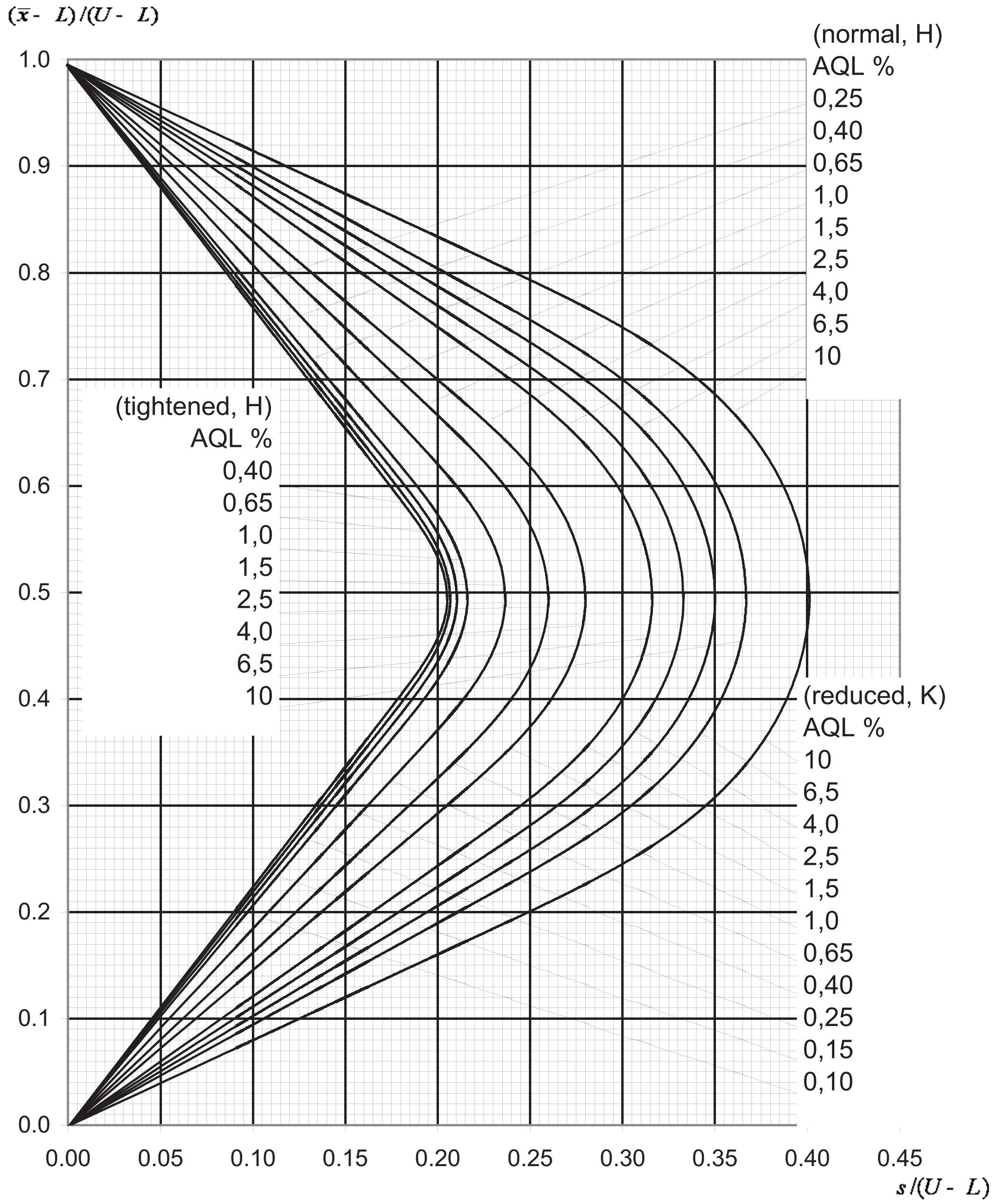


Figure 24 — Chart s-H: Acceptance curves for combined control of double specification limits for sample size code letter H under normal and tightened inspection and for sample size code letter K under reduced inspection

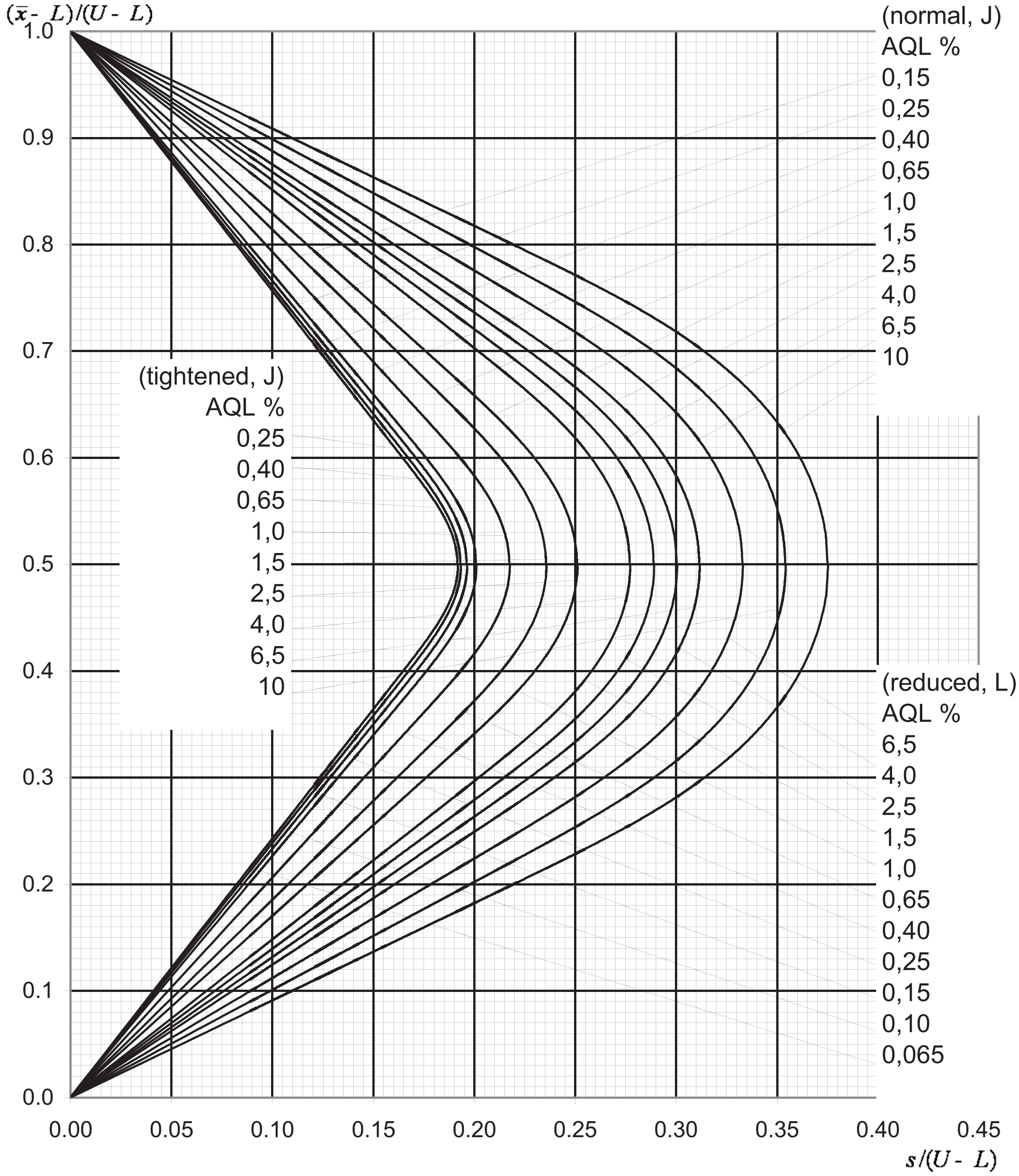


Figure 25 — Chart s-J: Acceptance curves for combined control of double specification limits for sample size code letter J under normal and tightened inspection and for sample size code letter L under reduced inspection

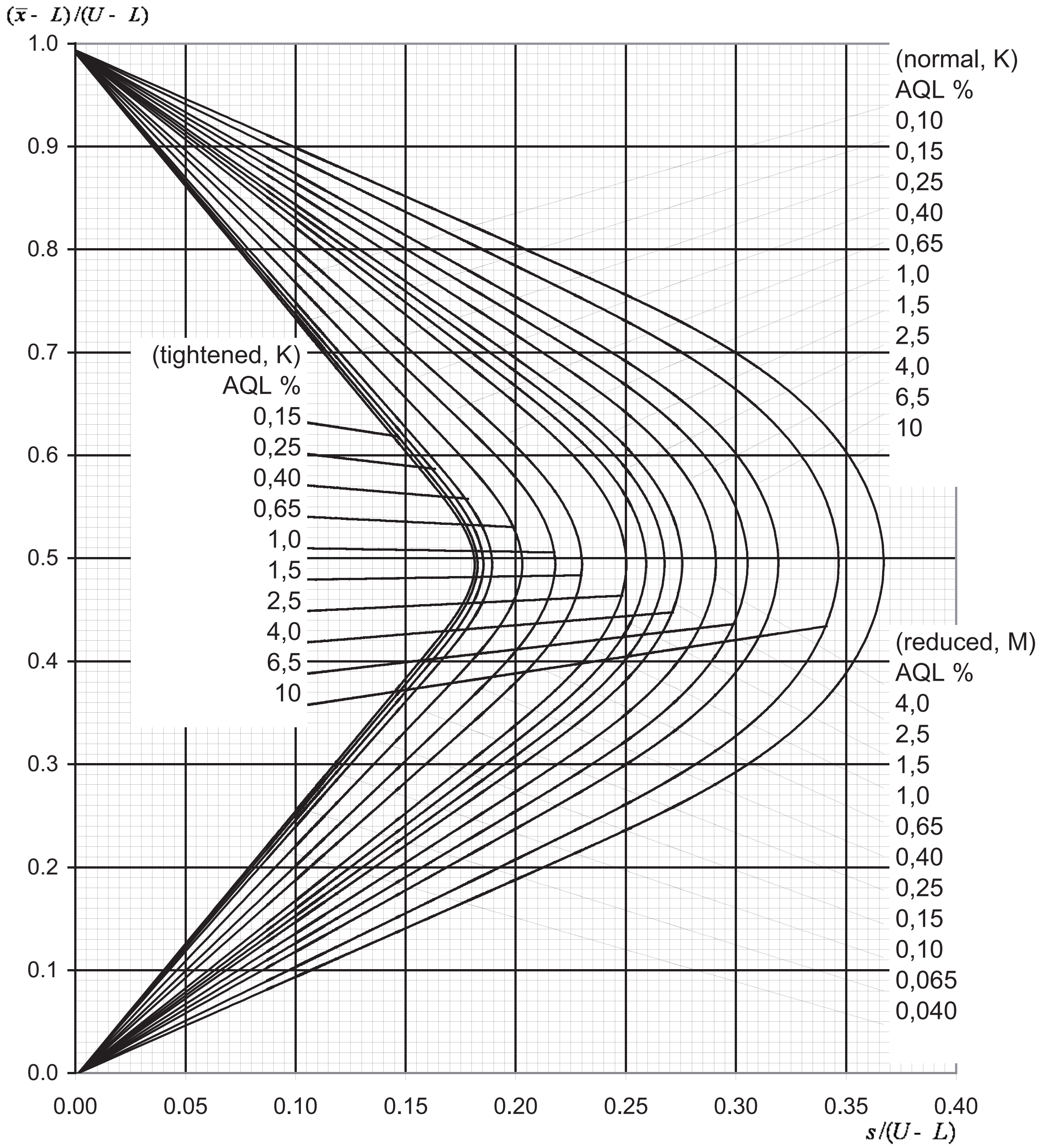


Figure 26 — Chart s-K: Acceptance curves for combined control of double specification limits for sample size code letter K under normal and tightened inspection and for sample size code letter M under reduced inspection

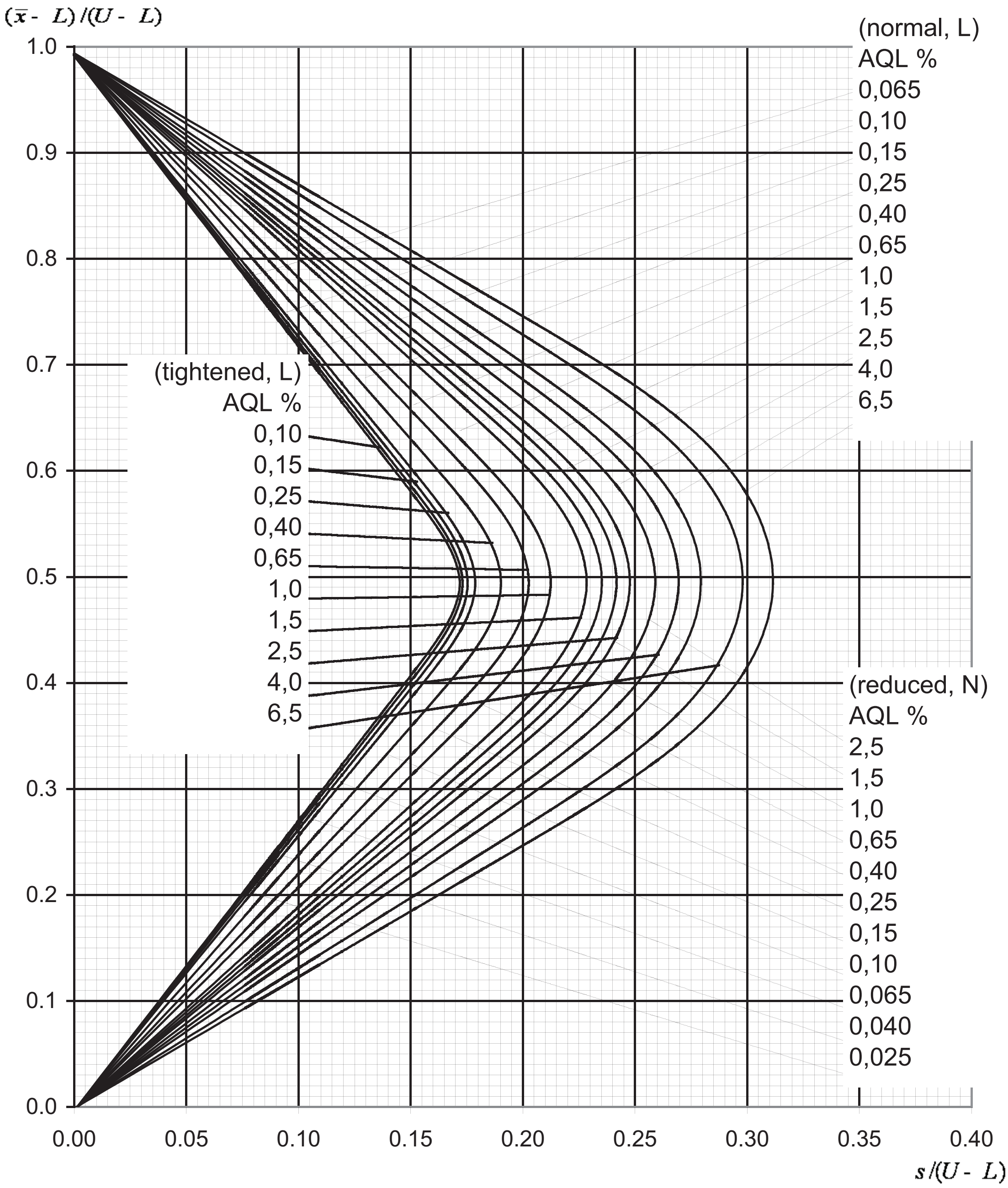


Figure 27 — Chart s-L: Acceptance curves for combined control of double specification limits for sample size code letter L under normal and tightened inspection and for sample size code letter N under reduced inspection

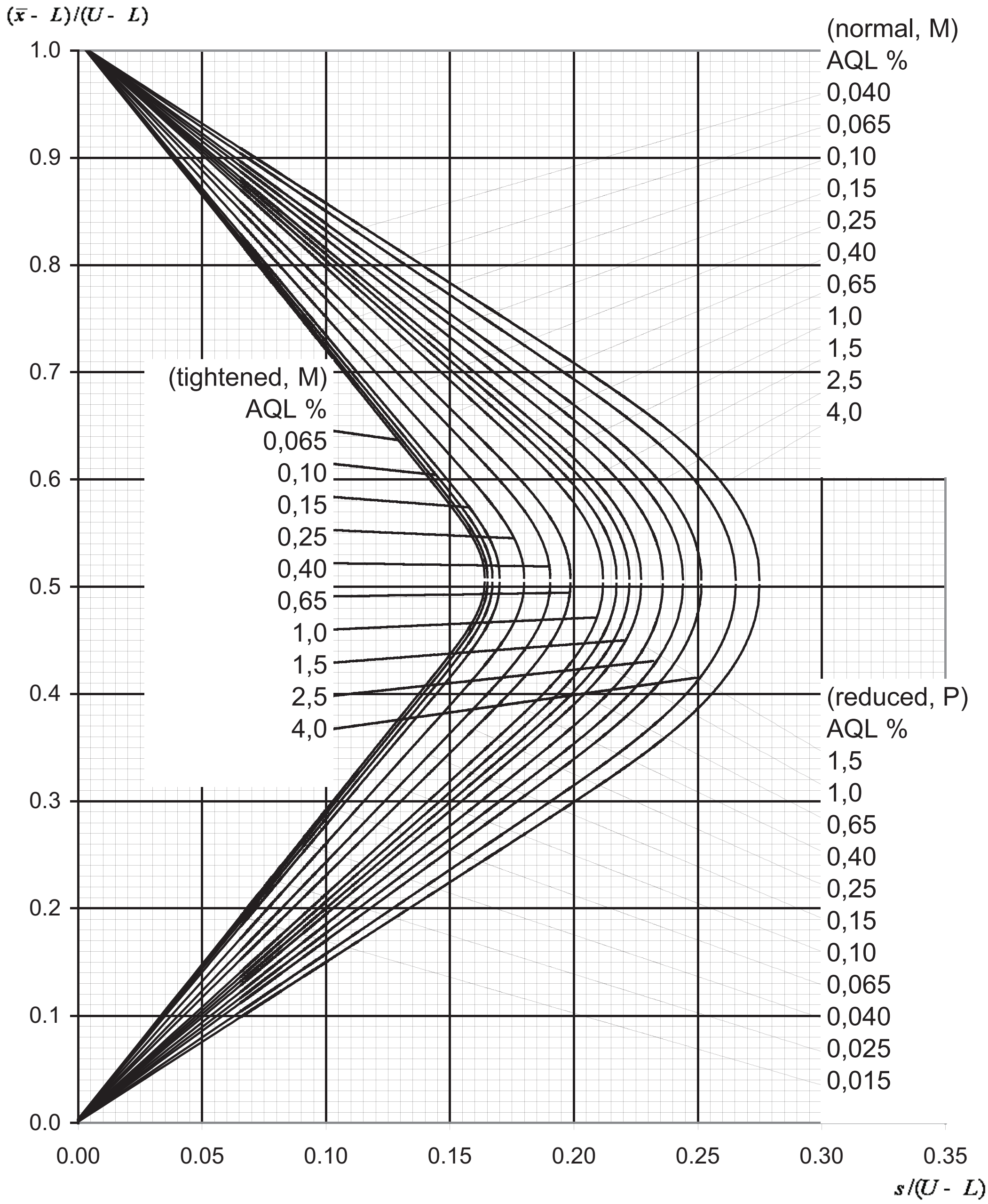


Figure 28 — Chart s-M: Acceptance curves for combined control of double specification limits for sample size code letter M under normal and tightened inspection and for sample size code letter P under reduced inspection

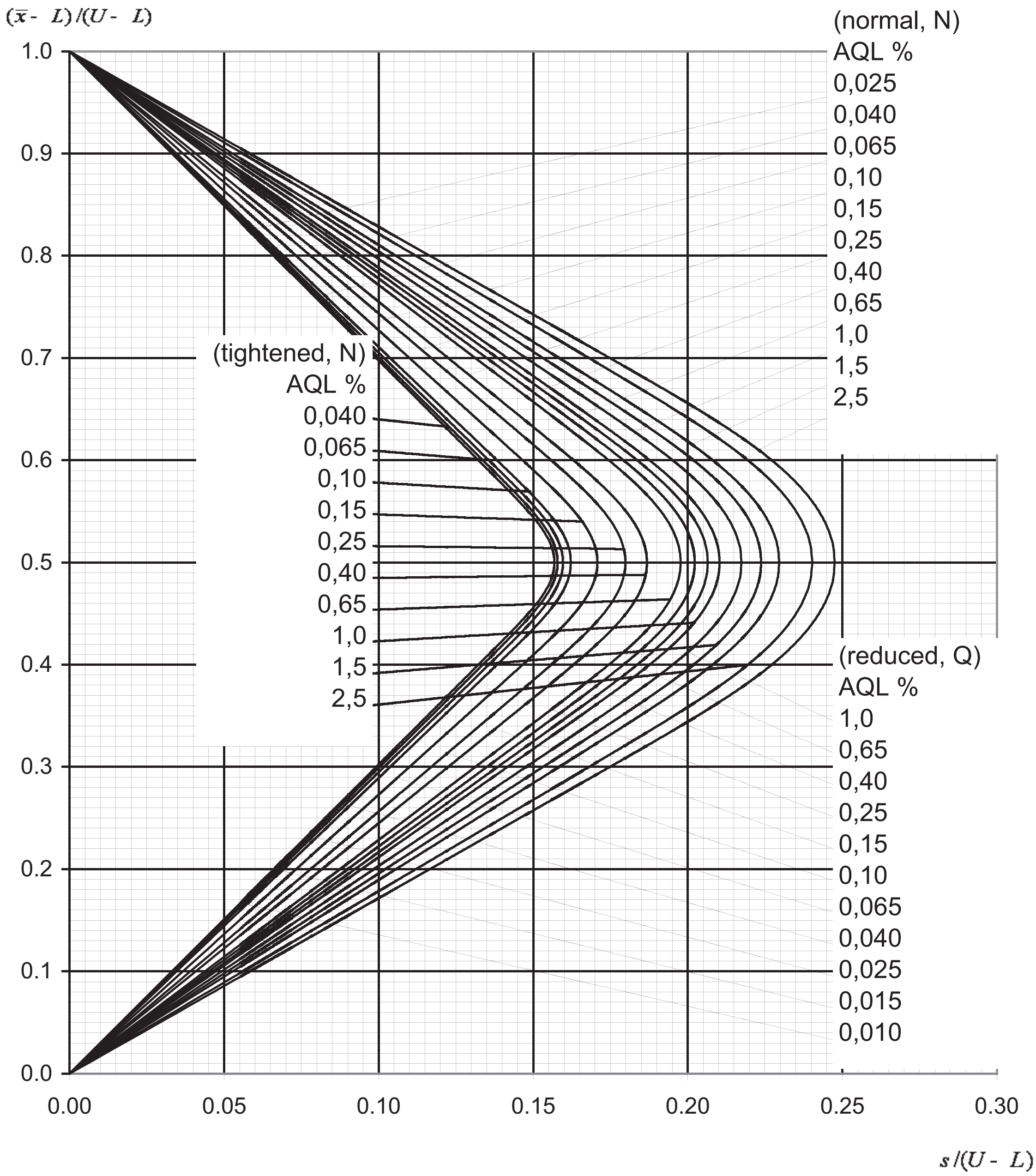


Figure 29 — Chart s-N: Acceptance curves for combined control of double specification limits for sample size code letter N under normal and tightened inspection and for sample size code letter Q under reduced inspection

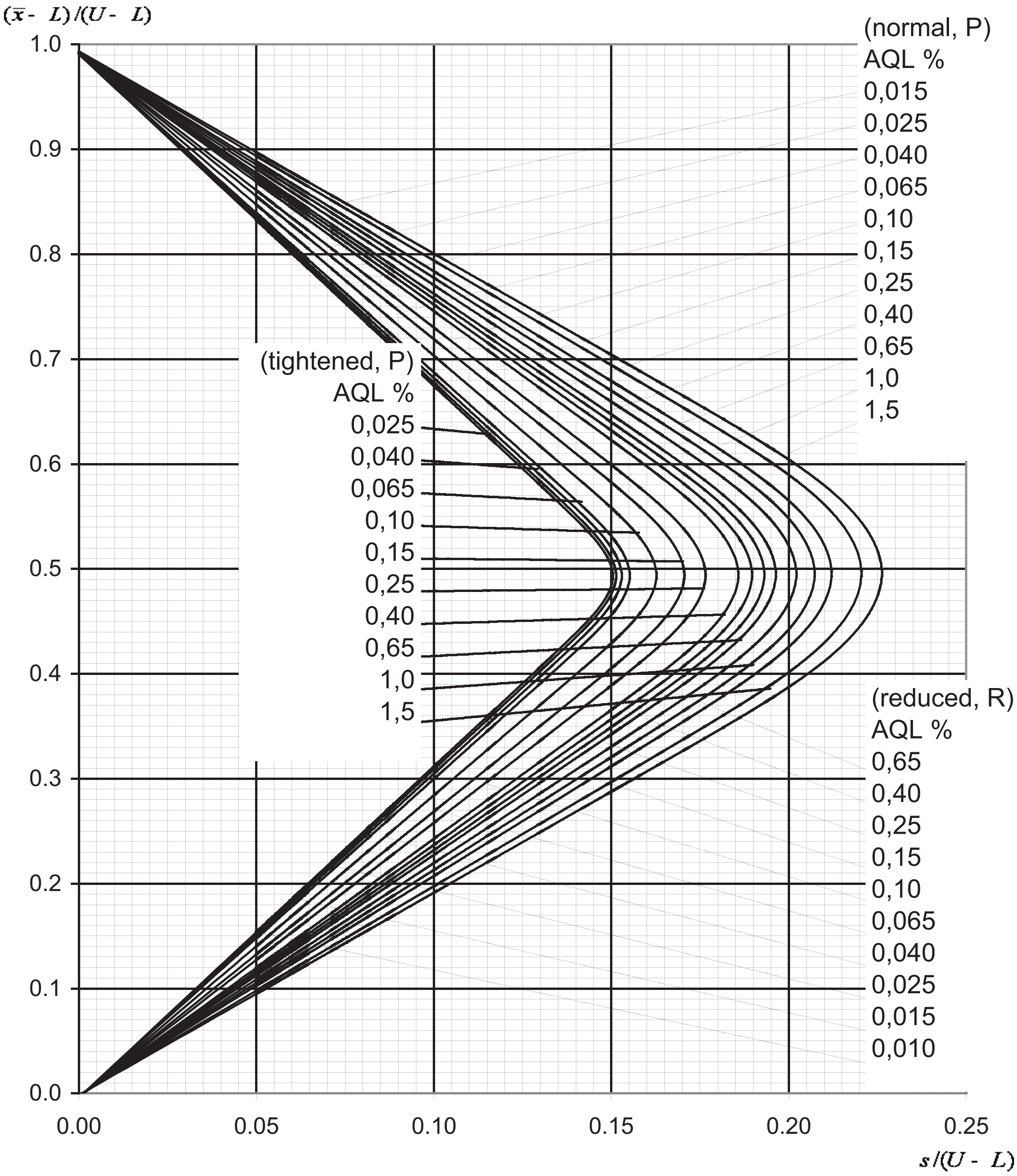


Figure 30 — Chart s-P: Acceptance curves for combined control of double specification limits for sample size code letter P under normal and tightened inspection and for sample size code letter R under reduced inspection

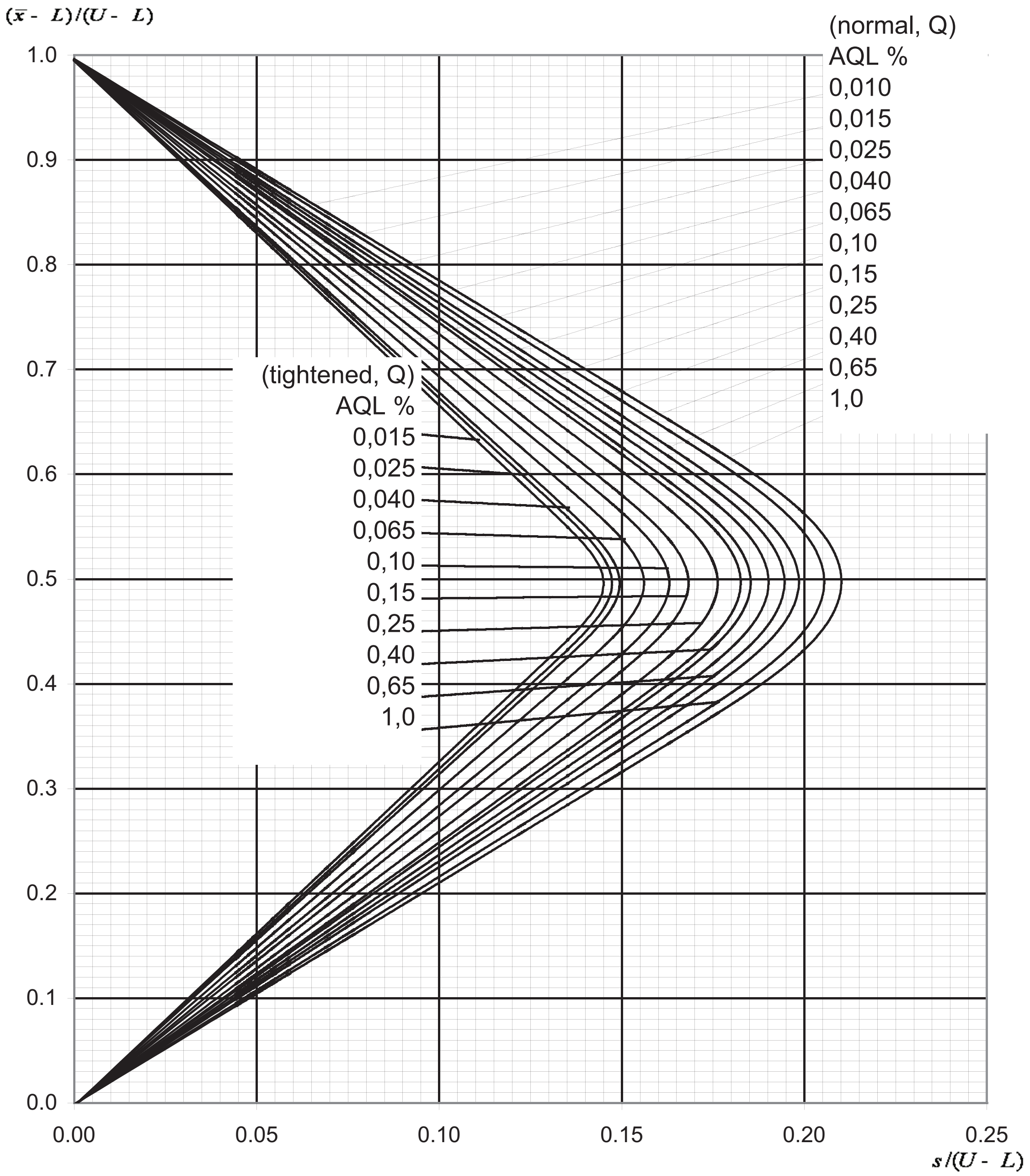


Figure 31 — Chart s-Q: Acceptance curves for combined control of double specification limits for sample size code letter Q under normal and tightened inspection

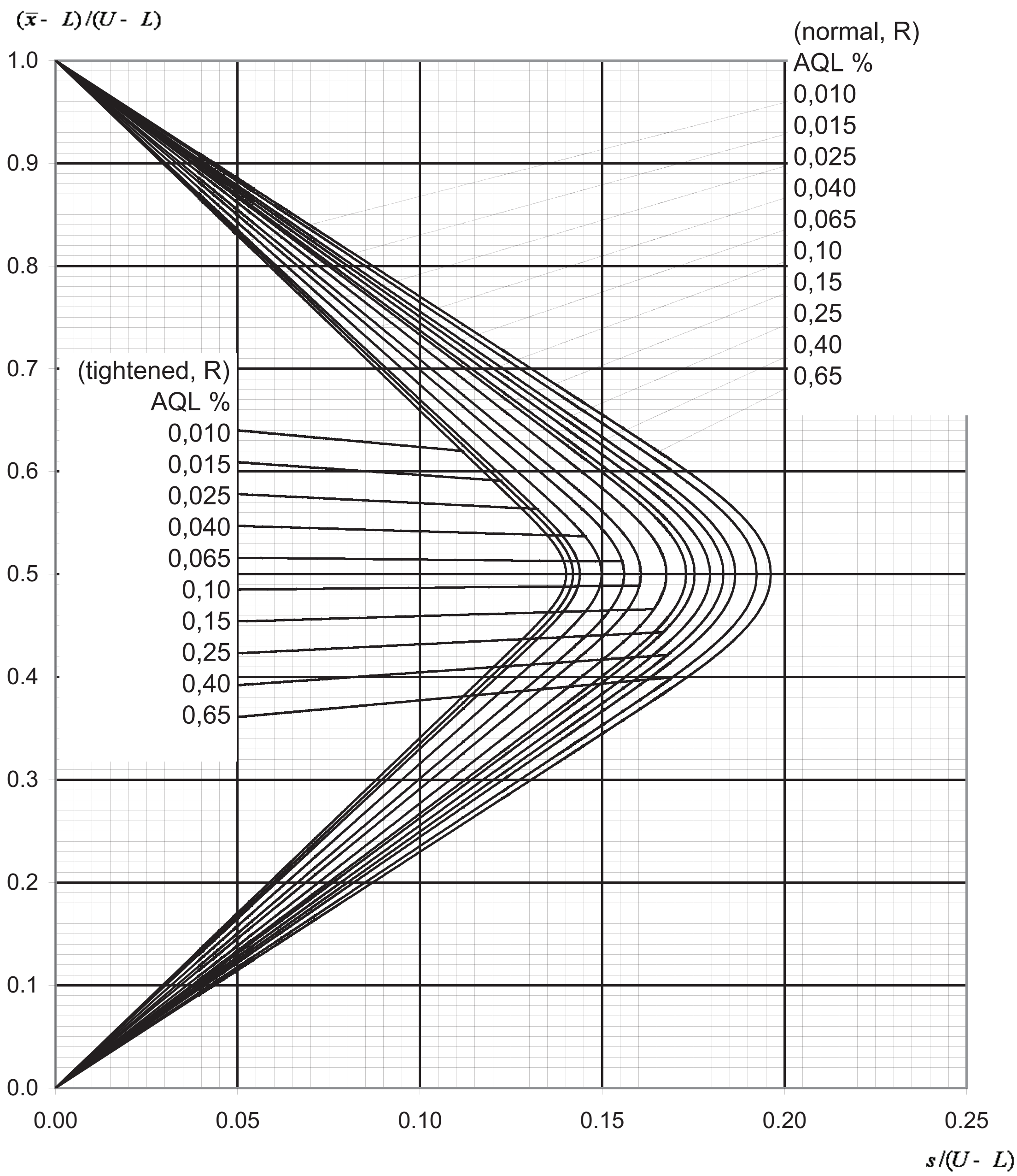


Figure 32 — Chart s-R: Acceptance curves for combined control of double specification limits for sample size code letter R under normal and tightened inspection

Annex A
(normative)

Table for determining the sample size code letter

Table A.1 — Sample size code letters and inspection levels

Lot or batch size	Special inspection levels				General inspection levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	B	B	B	B	B	B	B
9 to 15	B	B	B	B	B	B	C
16 to 25	B	B	B	B	B	C	D
26 to 50	B	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1 200	C	C	E	F	G	J	K
1 201 to 3 200	C	D	E	G	H	K	L
3 201 to 10 000	C	D	F	G	J	L	M
10 001 to 35 000	C	D	F	H	K	M	N
35 001 to 150 000	D	E	G	J	L	N	P
150 001 to 500 000	D	E	G	J	M	P	Q
500 000 and over	D	E	H	K	N	Q	R

NOTE The sample size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

Annex B
(normative)

Form *k* for single sampling plans: *s*-method

Table B.1 — Single sampling plans of Form *k* for normal inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,735	4 0,586
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,061	6 0,939	5 0,550
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	9 1,323	9 1,218	6 0,887	7 0,507
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	13 1,569	13 1,475	9 1,190	9 0,869	9 0,618
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,889	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935	14 0,601
G	↓	↓	↓	↓	↓	↓	↓	↓	15 2,079	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945	21 0,724
H	↓	↓	↓	↓	↓	↓	↓	18 2,254	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 1,036	33 0,806
J	↓	↓	↓	↓	↓	↓	23 2,425	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	49 1,316	52 1,120	53 0,911
K	↓	↓	↓	↓	↓	28 2,580	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	69 1,552	75 1,377	79 1,195	82 0,946
L	↓	↓	↓	↓	34 2,737	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	94 1,777	105 1,619	115 1,456	124 1,239	↑
M	↓	↓	↓	40 2,882	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	124 1,977	143 1,832	159 1,683	178 1,488	↑	↑
N	↓	↓	47 3,023	75 2,948	82 2,892	71 2,728	93 2,564	110 2,449	137 2,285	159 2,166	186 2,031	213 1,894	247 1,716	↑	↑	↑
P	↓	55 3,161	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	202 2,347	239 2,220	277 2,092	332 1,928	↑	↑	↑	↑
Q	63 3,288	101 3,219	110 3,167	102 3,016	132 2,867	159 2,762	207 2,615	244 2,508	293 2,388	348 2,268	424 2,114	↑	↑	↑	↑	↑
R	116 3,351	127 3,301	120 3,156	155 3,012	189 2,912	247 2,771	298 2,670	362 2,556	438 2,443	541 2,298	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.2 — Single sampling plans of Form *k* for tightened inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,735
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,061	6 0,939
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	9 1,323	9 1,218	6 0,887
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	13 1,569	13 1,475	9 1,190	9 0,869
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,889	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	15 2,079	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945
H	↓	↓	↓	↓	↓	↓	↓	↓	18 2,254	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 0,954
J	↓	↓	↓	↓	↓	↓	↓	23 2,425	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	50 1,245	53 1,010
K	↓	↓	↓	↓	↓	↓	28 2,580	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	71 1,489	78 1,281	82 1,045
L	↓	↓	↓	↓	↓	34 2,737	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	99 1,720	111 1,533	122 1,325	↑
M	↓	↓	↓	↓	40 2,882	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	131 1,924	150 1,752	170 1,564	↑	↑
N	↓	↓	↓	47 3,023	75 2,948	82 2,892	73 2,728	93 2,564	110 2,449	137 2,285	169 2,117	201 1,958	233 1,785	↑	↑	↑
P	↓	↓	55 3,161	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	214 2,300	260 2,152	312 1,992	↑	↑	↑	↑
Q	↓	63 3,288	101 3,219	110 3,167	102 3,016	132 2,867	159 2,762	207 2,615	262 2,464	323 2,324	395 2,174	↑	↑	↑	↑	↑
R	90 3,408	116 3,351	127 3,301	120 3,156	155 3,012	189 2,912	247 2,771	320 2,628	398 2,495	498 2,354	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.3 — Single sampling plans of Form *k* for reduced inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B – D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,850	4 0,735	4 0,586	7 0,218
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,155	6 1,061	6 0,939	5 0,550	9 0,162
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	8 1,406	9 1,323	9 1,218	6 0,887	7 0,507	8 0,231
G	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	11 1,642	13 1,569	13 1,475	9 1,190	9 0,869	9 0,618	12 0,237
H	↓	↓	↓	↓	↓	↓	↓	11 1,889	15 1,835	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935	14 0,601	13 0,454
J	↓	↓	↓	↓	↓	↓	15 2,079	19 2,033	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945	21 0,830	21 0,626
K	↓	↓	↓	↓	↓	18 2,254	24 2,209	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 1,126	33 0,954	33 0,806
L	↓	↓	↓	↓	23 2,425	30 2,385	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	48 1,394	50 1,245	52 1,120	↑
M	↓	↓	↓	28 2,580	37 2,543	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	66 1,622	71 1,489	75 1,377	↑	↑
N	↓	↓	34 2,737	44 2,701	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	90 1,842	99 1,720	105 1,619	↑	↑	↑
P	↓	40 2,882	52 2,848	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	117 2,037	131 1,924	143 1,832	↑	↑	↑	↑
Q	47 3,023	61 2,991	75 2,948	82 2,892	73 2,728	93 2,564	110 2,449	137 2,285	149 2,222	169 2,117	186 2,031	↑	↑	↑	↑	↑
R	71 3,131	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	187 2,399	214 2,300	239 2,220	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex C
(normative)

Form *k* for single sampling plans: *σ*-method

Table C.1 — Single sampling plans of Form *k* for normal inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,571	3 0,417
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 0,945	5 0,821	4 0,436
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	6 1,240	6 1,128	5 0,770	5 0,431
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	7 1,506	8 1,419	7 1,115	7 0,792	7 0,555
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,845	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877	11 0,564
G	↓	↓	↓	↓	↓	↓	↓	↓	5 2,006	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906	15 0,694
H	↓	↓	↓	↓	↓	↓	↓	6 2,218	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	19 1,009	23 0,786
J	↓	↓	↓	↓	↓	↓	7 2,401	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	24 1,293	29 1,102	34 0,897
K	↓	↓	↓	↓	↓	7 2,541	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	29 1,533	35 1,361	42 1,182	53 0,937
L	↓	↓	↓	↓	8 2,710	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	34 1,761	42 1,606	52 1,446	66 1,231	↑
M	↓	↓	↓	8 2,844	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	39 1,963	50 1,821	61 1,674	79 1,481	↑	↑
N	↓	↓	9 2,996	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	45 2,155	57 2,022	72 1,887	94 1,710	↑	↑	↑
P	↓	10 3,141	17 3,069	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	51 2,337	65 2,212	82 2,086	110 1,923	↑	↑	↑	↑
Q	11 3,275	18 3,207	20 3,155	20 3,002	28 2,856	35 2,752	49 2,607	57 2,500	72 2,381	92 2,262	125 2,110	↑	↑	↑	↑	↑
R	19 3,339	21 3,289	22 3,145	30 3,002	38 2,903	54 2,764	64 2,663	81 2,550	105 2,438	142 2,294	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.2 — Single sampling plans of Form *k* for tightened inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,571
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 0,945	5 0,821
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	6 1,240	6 1,128	5 0,770
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	7 1,506	8 1,419	7 1,115	7 0,792
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,845	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 2,006	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906
H	↓	↓	↓	↓	↓	↓	↓	↓	6 2,218	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	20 0,929
J	↓	↓	↓	↓	↓	↓	↓	7 2,401	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	25 1,223	32 0,994
K	↓	↓	↓	↓	↓	↓	7 2,541	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	31 1,471	39 1,267	49 1,035
L	↓	↓	↓	↓	↓	8 2,710	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	37 1,705	47 1,521	61 1,316	↑
M	↓	↓	↓	↓	8 2,844	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	43 1,912	55 1,742	72 1,556	↑	↑
N	↓	↓	↓	9 2,996	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	49 2,106	65 1,950	85 1,779	↑	↑	↑
P	↓	↓	10 3,142	17 3,076	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	55 2,291	74 2,145	99 1,987	↑	↑	↑	↑
Q	↓	11 3,275	18 3,207	20 3,155	20 3,002	28 2,856	35 2,752	49 2,607	61 2,456	83 2,318	112 2,169	↑	↑	↑	↑	↑
R	14 3,391	19 3,339	21 3,289	22 3,145	30 3,002	38 2,903	54 2,764	68 2,621	92 2,490	126 2,350	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.3 — Single sampling plans of Form *k* for reduced inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B – D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,679	4 0,571	3 0,417	6 0,187
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 1,047	5 0,945	5 0,821	4 0,436	8 0,145
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	5 1,314	6 1,240	6 1,128	5 0,770	5 0,431	7 0,204
G	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	6 1,581	7 1,506	8 1,419	7 1,115	7 0,792	7 0,555	11 0,220
H	↓	↓	↓	↓	↓	↓	↓	5 1,845	7 1,788	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877	11 0,564	11 0,424
J	↓	↓	↓	↓	↓	↓	5 2,006	7 1,982	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906	14 0,796	16 0,601
K	↓	↓	↓	↓	↓	6 2,218	8 2,171	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	18 1,096	20 0,929	23 0,786
L	↓	↓	↓	↓	7 2,401	9 2,355	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	22 1,369	25 1,223	29 1,102	↑
M	↓	↓	↓	7 2,541	10 2,518	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	26 1,601	31 1,471	35 1,361	↑	↑
N	↓	↓	8 2,710	10 2,669	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	31 1,825	37 1,705	42 1,606	↑	↑	↑
P	↓	8 2,844	11 2,822	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	38 2,024	43 1,912	50 1,821	↑	↑	↑	↑
Q	9 2,996	12 2,969	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	45 2,212	49 2,106	57 2,022	↑	↑	↑	↑	↑
R	13 3,113	17 3,076	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	50 2,390	55 2,291	65 2,212	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-2.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex D
(normative)

Values of f_S for maximum sample standard deviation (MSSD)

Table D.1 — Values of f_S for maximum sample standard deviation for combined control of double specification limits: normal inspection, s–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B													↓	0,475	0,447	0,479
C												↓	0,365	0,366	0,388	0,484
D											↓	0,303	0,312	0,328	0,399	0,494
E										↓	0,265	0,274	0,285	0,333	0,395	0,458
F									↓	0,241	0,248	0,257	0,292	0,334	0,375	0,461
G								↓	0,221	0,227	0,234	0,260	0,290	0,318	0,371	0,424
H							↓	0,206	0,211	0,216	0,237	0,260	0,280	0,316	0,350	0,401
J						↓	0,192	0,197	0,201	0,218	0,236	0,251	0,277	0,301	0,333	0,376
K					↓	0,182	0,185	0,189	0,203	0,218	0,230	0,250	0,268	0,291	0,319	0,367
L				↓	0,172	0,175	0,179	0,190	0,203	0,212	0,229	0,242	0,259	0,279	0,312	↑
M			↓	0,164	0,167	0,170	0,180	0,190	0,199	0,212	0,222	0,236	0,251	0,275	↑	
N		↓	0,157	0,160	0,162	0,171	0,180	0,187	0,198	0,206	0,217	0,230	0,248	↑		
P	↓	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,193	0,202	0,212	0,226	↑			
Q	0,145	0,147	0,149	0,156	0,163	0,168	0,176	0,183	0,190	0,199	0,210	↑				
R	0,142	0,144	0,150	0,156	0,161	0,168	0,173	0,180	0,187	0,196	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD, f_S, by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MSSD = $s_{max} = (U - L)f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.2 — Values of f_s for maximum sample standard deviation for combined control of double specification limits: tightened inspection, *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B														↓	0,475	0,447
C													↓	0,365	0,366	0,388
D												↓	0,303	0,312	0,328	0,399
E											↓	0,265	0,274	0,285	0,333	0,395
F										↓	0,241	0,248	0,257	0,292	0,334	0,375
G									↓	0,221	0,227	0,234	0,260	0,290	0,318	0,371
H								↓	0,206	0,211	0,216	0,237	0,260	0,280	0,316	0,367
J							↓	0,192	0,197	0,201	0,218	0,236	0,251	0,277	0,312	0,354
K						↓	0,182	0,185	0,189	0,203	0,218	0,230	0,250	0,276	0,305	0,347
L					↓	0,172	0,175	0,179	0,190	0,203	0,212	0,229	0,248	0,269	0,298	↑
M				↓	0,164	0,167	0,170	0,180	0,190	0,199	0,212	0,227	0,244	0,265	↑	
N			↓	0,157	0,160	0,162	0,171	0,180	0,187	0,198	0,210	0,224	0,240	↑		
P		↓	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,196	0,207	0,221	↑			
Q	↓	0,145	0,147	0,149	0,156	0,163	0,168	0,176	0,185	0,195	0,206	↑				
R	0,140	0,142	0,144	0,150	0,156	0,161	0,168	0,175	0,183	0,192	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD, f_S, by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MSSD = $s_{max} = (U - L)f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.3 — Values of f_S for maximum sample standard deviation for combined control of double specification limits: reduced inspection, *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B-D												0,475	0,426	0,447	0,479	0,602
E											0,365	0,350	0,366	0,388	0,484	0,632
F										0,303	0,303	0,312	0,328	0,399	0,494	0,598
G									0,265	0,267	0,274	0,285	0,333	0,395	0,458	0,599
H								0,241	0,243	0,248	0,257	0,292	0,334	0,375	0,461	0,510
J							0,221	0,223	0,227	0,234	0,260	0,290	0,318	0,371	0,397	0,452
K						0,206	0,207	0,211	0,216	0,237	0,260	0,280	0,316	0,333	0,367	0,401
L					0,192	0,194	0,197	0,202	0,218	0,233	0,251	0,277	0,289	0,312	0,333	
M				0,182	0,183	0,185	0,189	0,203	0,218	0,230	0,250	0,259	0,276	0,291		
N			0,172	0,173	0,175	0,179	0,190	0,203	0,212	0,229	0,235	0,248	0,259			
P		0,164	0,165	0,167	0,170	0,180	0,190	0,199	0,212	0,217	0,227	0,236				
Q	0,157	0,158	0,160	0,162	0,171	0,180	0,187	0,198	0,202	0,210	0,217					
R	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,190	0,196	0,202						

NOTE The MSSD is obtained by multiplying the standardized MSSD, f_S, by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MSSD = $s_{max} = (U - L)f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under reduced inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, there is a possibility, but not a certainty, that the lot will be accepted.

Annex E
(normative)

Values of f_σ for maximum process standard deviation (MPSD)

Table E.1 — Values of f_σ for maximum process standard deviation for combined control of double specification limits: σ–method

Acceptance quality limit (in percent nonconforming)	f_σ
0,010 7	0,125
0,015	0,129
0,025	0,132
0,040	0,137
0,065	0,141
0,10	0,147
0,15	0,152
0,25	0,157
0,40	0,165
0,65	0,174
1,0	0,184
1,5	0,194
2,5	0,206
4,0	0,223
6,5	0,243
10	0,271

NOTE The MPSD is obtained by multiplying the standardized MPSD, f_σ, by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MPSD = $(U - L)f_\sigma$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for combined control of double specification limits when the process variability is known. If the process standard deviation is less than the MPSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Annex F
(normative)

Estimating the process fraction nonconforming for sample size 3: *s*-method

Table F.1 — Estimated process fraction nonconforming, $\hat{p}$, as a function of the quality statistic Q

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,00	0,500 0	0,499 7	0,499 4	0,499 0	0,498 7	0,498 4	0,498 1	0,497 8	0,497 5	0,497 1
	0,01	0,496 8	0,496 5	0,496 2	0,495 9	0,495 5	0,495 2	0,494 9	0,494 6	0,494 3	0,494 0
	0,02	0,493 6	0,493 3	0,493 0	0,492 7	0,492 4	0,492 0	0,491 7	0,491 4	0,491 1	0,490 8
	0,03	0,490 4	0,490 1	0,489 8	0,489 5	0,489 2	0,488 9	0,488 5	0,488 2	0,487 9	0,487 6
	0,04	0,487 3	0,486 9	0,486 6	0,486 3	0,486 0	0,485 7	0,485 4	0,485 0	0,484 7	0,484 4
	0,05	0,484 1	0,483 8	0,483 4	0,483 1	0,482 8	0,482 5	0,482 2	0,481 8	0,481 5	0,481 2
	0,06	0,480 9	0,480 6	0,480 3	0,479 9	0,479 6	0,479 3	0,479 0	0,478 7	0,478 3	0,478 0
	0,07	0,477 7	0,477 4	0,477 1	0,476 7	0,476 4	0,476 1	0,475 8	0,475 5	0,475 1	0,474 8
	0,08	0,474 5	0,474 2	0,473 9	0,473 5	0,473 2	0,472 9	0,472 6	0,472 3	0,472 0	0,471 6
	0,09	0,471 3	0,471 0	0,470 7	0,470 4	0,470 0	0,469 7	0,469 4	0,469 1	0,468 8	0,468 4
	0,10	0,468 1	0,467 8	0,467 5	0,467 2	0,466 8	0,466 5	0,466 2	0,465 9	0,465 6	0,465 2
	0,11	0,464 9	0,464 6	0,464 3	0,464 0	0,463 6	0,463 3	0,463 0	0,462 7	0,462 4	0,462 0
	0,12	0,461 7	0,461 4	0,461 1	0,460 7	0,460 4	0,460 1	0,459 8	0,459 5	0,459 1	0,458 8
	0,13	0,458 5	0,458 2	0,457 9	0,457 5	0,457 2	0,456 9	0,456 6	0,456 3	0,455 9	0,455 6
	0,14	0,455 3	0,455 0	0,454 6	0,454 3	0,454 0	0,453 7	0,453 4	0,453 0	0,452 7	0,452 4
	0,15	0,452 1	0,451 8	0,451 4	0,451 1	0,450 8	0,450 5	0,450 1	0,449 8	0,449 5	0,449 2
	0,16	0,448 9	0,448 5	0,448 2	0,447 9	0,447 6	0,447 2	0,446 9	0,446 6	0,446 3	0,445 9
	0,17	0,445 6	0,445 3	0,445 0	0,444 7	0,444 3	0,444 0	0,443 7	0,443 4	0,443 0	0,442 7
	0,18	0,442 4	0,442 1	0,441 7	0,441 4	0,441 1	0,440 8	0,440 4	0,440 1	0,439 8	0,439 5
	0,19	0,439 2	0,438 8	0,438 5	0,438 2	0,437 9	0,437 5	0,437 2	0,436 9	0,436 6	0,436 2
	0,20	0,435 9	0,435 6	0,435 3	0,434 9	0,434 6	0,434 3	0,434 0	0,433 6	0,433 3	0,433 0
	0,21	0,432 7	0,432 3	0,432 0	0,431 7	0,431 4	0,431 0	0,430 7	0,430 4	0,430 0	0,429 7
	0,22	0,429 4	0,429 1	0,428 7	0,428 4	0,428 1	0,427 8	0,427 4	0,427 1	0,426 8	0,426 5
	0,23	0,426 1	0,425 8	0,425 5	0,425 1	0,424 8	0,424 5	0,424 2	0,423 8	0,423 5	0,423 2
	0,24	0,422 9	0,422 5	0,422 2	0,421 9	0,421 5	0,421 2	0,420 9	0,420 6	0,420 2	0,419 9
	0,25	0,419 6	0,419 2	0,418 9	0,418 6	0,418 3	0,417 9	0,417 6	0,417 3	0,416 9	0,416 6
	0,26	0,416 3	0,415 9	0,415 6	0,415 3	0,415 0	0,414 6	0,414 3	0,414 0	0,413 6	0,413 3
	0,27	0,413 0	0,412 6	0,412 3	0,412 0	0,411 7	0,411 3	0,411 0	0,410 7	0,410 3	0,410 0
	0,28	0,409 7	0,409 3	0,409 0	0,408 7	0,408 3	0,408 0	0,407 7	0,407 3	0,407 0	0,406 7

NOTE For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Table F.1 — *(continued)*

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,29	0,406 3	0,406 0	0,405 7	0,405 3	0,405 0	0,404 7	0,404 3	0,404 0	0,403 7	0,403 3
	0,30	0,403 0	0,402 7	0,402 3	0,402 0	0,401 7	0,401 3	0,401 0	0,400 7	0,400 3	0,400 0
	0,31	0,399 7	0,399 3	0,399 0	0,398 7	0,398 3	0,398 0	0,397 7	0,397 3	0,397 0	0,396 7
	0,32	0,396 3	0,396 0	0,395 6	0,395 3	0,395 0	0,394 6	0,394 3	0,394 0	0,393 6	0,393 3
	0,33	0,393 0	0,392 6	0,392 3	0,391 9	0,391 6	0,391 3	0,390 9	0,390 6	0,390 2	0,389 9
	0,34	0,389 6	0,389 2	0,388 9	0,388 6	0,388 2	0,387 9	0,387 5	0,387 2	0,386 9	0,386 5
	0,35	0,386 2	0,385 8	0,385 5	0,385 2	0,384 8	0,384 5	0,384 1	0,383 8	0,383 5	0,383 1
	0,36	0,382 8	0,382 4	0,382 1	0,381 8	0,381 4	0,381 1	0,380 7	0,380 4	0,380 0	0,379 7
	0,37	0,379 4	0,379 0	0,378 7	0,378 3	0,378 0	0,377 6	0,377 3	0,377 0	0,376 6	0,376 3
	0,38	0,375 9	0,375 6	0,375 2	0,374 9	0,374 5	0,374 2	0,373 9	0,373 5	0,373 2	0,372 8
	0,39	0,372 5	0,372 1	0,371 8	0,371 4	0,371 1	0,370 7	0,370 4	0,370 1	0,369 7	0,369 4
	0,40	0,369 0	0,368 7	0,368 3	0,368	0,367 6	0,367 3	0,366 9	0,366 6	0,366 2	0,365 9
	0,41	0,365 5	0,365 2	0,364 8	0,364 5	0,364 1	0,363 8	0,363 4	0,363 1	0,362 7	0,362 4
	0,42	0,362 0	0,361 7	0,361 3	0,361 0	0,360 6	0,360 3	0,359 9	0,359 6	0,359 2	0,358 9
	0,43	0,358 5	0,358 2	0,357 8	0,357 5	0,357 1	0,356 7	0,356 4	0,356	0,355 7	0,355 3
	0,44	0,355 0	0,354 6	0,354 3	0,353 9	0,353 6	0,353 2	0,352 8	0,352 5	0,352 1	0,351 8
	0,45	0,351 4	0,351 1	0,350 7	0,350 4	0,350 0	0,349 6	0,349 3	0,348 9	0,348 6	0,348 2
	0,46	0,347 8	0,347 5	0,347 1	0,346 8	0,346 4	0,346 1	0,345 7	0,345 3	0,345 0	0,344 6
	0,47	0,344 3	0,343 9	0,343 5	0,343 2	0,342 8	0,342 4	0,342 1	0,341 7	0,341 4	0,341 0
	0,48	0,340 6	0,340 3	0,339 9	0,339 5	0,339 2	0,338 8	0,338 5	0,338 1	0,337 7	0,337 4
	0,49	0,337 0	0,336 6	0,336 3	0,335 9	0,335 5	0,335 2	0,334 8	0,334 4	0,334 1	0,333 7
	0,50	0,333 3	0,333 0	0,332 6	0,332 2	0,331 9	0,331 5	0,331 1	0,330 8	0,330 4	0,330 0
	0,51	0,329 6	0,329 3	0,328 9	0,328 5	0,328 2	0,327 8	0,327 4	0,327 0	0,326 7	0,326 3
	0,52	0,325 9	0,325 6	0,325 2	0,324 8	0,324 4	0,324 1	0,323 7	0,323 3	0,322 9	0,322 6
	0,53	0,322 2	0,321 8	0,321 4	0,321 1	0,320 7	0,320 3	0,319 9	0,319 6	0,319 2	0,318 8
	0,54	0,318 4	0,318 0	0,317 7	0,317 3	0,316 9	0,316 5	0,316 1	0,315 8	0,315 4	0,315 0
	0,55	0,314 6	0,314 2	0,313 9	0,313 5	0,313 1	0,312 7	0,312 3	0,312 0	0,311 6	0,311 2
	0,56	0,310 8	0,310 4	0,310 0	0,309 6	0,309 3	0,308 9	0,308 5	0,308 1	0,307 7	0,307 3
	0,57	0,306 9	0,306 6	0,306 2	0,305 8	0,305 4	0,305 0	0,304 6	0,304 2	0,303 8	0,303 4
	0,58	0,303 1	0,302 7	0,302 3	0,301 9	0,301 5	0,301 1	0,300 7	0,300 3	0,299 9	0,299 5
	0,59	0,299 1	0,298 7	0,298 3	0,297 9	0,297 5	0,297 2	0,296 8	0,296 4	0,296 0	0,295 6
	0,60	0,295 2	0,294 8	0,294 4	0,294 0	0,293 6	0,293 2	0,292 8	0,292 4	0,292 0	0,291 6
	0,61	0,291 2	0,290 8	0,290 4	0,290 0	0,289 6	0,289 2	0,288 8	0,288 3	0,287 9	0,287 5
	0,62	0,287 1	0,286 7	0,286 3	0,285 9	0,285 5	0,285 1	0,284 7	0,284 3	0,283 9	0,283 5
	0,63	0,283 1	0,282 6	0,282 2	0,281 8	0,281 4	0,281 0	0,280 6	0,280 2	0,279 8	0,279 3
	0,64	0,278 9	0,278 5	0,278 1	0,277 7	0,277 3	0,276 9	0,276 4	0,276 0	0,275 6	0,275 2

NOTE For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Table F.1 — *(continued)*

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,65	0,274 8	0,274 3	0,273 9	0,273 5	0,273 1	0,272 7	0,272 2	0,271 8	0,271 4	0,271 0
	0,66	0,270 6	0,270 1	0,269 7	0,269 3	0,268 9	0,268 4	0,268 0	0,267 6	0,267 2	0,266 7
	0,67	0,266 3	0,265 9	0,265 4	0,265 0	0,264 6	0,264 1	0,263 7	0,263 3	0,262 8	0,262 4
	0,68	0,262 0	0,261 5	0,261 1	0,260 7	0,260 2	0,259 8	0,259 4	0,258 9	0,258 5	0,258 0
	0,69	0,257 6	0,257 2	0,256 7	0,256 3	0,255 8	0,255 4	0,255 0	0,254 5	0,254 1	0,253 6
	0,70	0,253 2	0,252 7	0,252 3	0,251 8	0,251 4	0,250 9	0,250 5	0,250 0	0,249 6	0,249 1
	0,71	0,248 7	0,248 2	0,247 8	0,247 3	0,246 9	0,246 4	0,246 0	0,245 5	0,245 1	0,244 6
	0,72	0,244 1	0,243 7	0,243 2	0,242 8	0,242 3	0,241 8	0,241 4	0,240 9	0,240 5	0,240 0
	0,73	0,239 5	0,239 1	0,238 6	0,238 1	0,237 7	0,237 2	0,236 7	0,236 2	0,235 8	0,235 3
	0,74	0,234 8	0,234 4	0,233 9	0,233 4	0,232 9	0,232 4	0,232 0	0,231 5	0,231 0	0,230 5
	0,75	0,230 1	0,229 6	0,229 1	0,228 6	0,228 1	0,227 6	0,227 2	0,226 7	0,226 2	0,225 7
	0,76	0,225 2	0,224 7	0,224 2	0,223 7	0,223 2	0,222 7	0,222 2	0,221 7	0,221 3	0,220 8
	0,77	0,220 3	0,219 8	0,219 3	0,218 8	0,218 3	0,217 7	0,217 2	0,216 7	0,216 2	0,215 7
	0,78	0,215 2	0,214 7	0,214 2	0,213 7	0,213 2	0,212 7	0,212 1	0,211 6	0,211 1	0,210 6
	0,79	0,210 1	0,209 6	0,209 0	0,208 5	0,208 0	0,207 5	0,206 9	0,206 4	0,205 9	0,205 4
	0,80	0,204 8	0,204 3	0,203 8	0,203 2	0,202 7	0,202 2	0,201 6	0,201 1	0,200 6	0,200 0
	0,81	0,199 5	0,198 9	0,198 4	0,197 8	0,197 3	0,196 7	0,196 2	0,195 6	0,195 1	0,194 5
	0,82	0,194 0	0,193 4	0,192 9	0,192 3	0,191 7	0,191 2	0,190 6	0,190 0	0,189 5	0,188 9
	0,83	0,188 3	0,187 8	0,187 2	0,186 6	0,186 0	0,185 5	0,184 9	0,184 3	0,183 7	0,183 1
	0,84	0,182 6	0,182 0	0,181 4	0,180 8	0,180 2	0,179 6	0,179 0	0,178 4	0,177 8	0,177 2
	0,85	0,176 6	0,176 0	0,175 4	0,174 8	0,174 2	0,173 6	0,172 9	0,172 3	0,171 7	0,171 1
	0,86	0,170 5	0,169 8	0,169 2	0,168 6	0,168 0	0,167 3	0,166 7	0,166 0	0,165 4	0,164 8
	0,87	0,164 1	0,163 5	0,162 8	0,162 2	0,161 5	0,160 9	0,160 2	0,159 5	0,158 9	0,158 2
	0,88	0,157 5	0,156 9	0,156 2	0,155 5	0,154 8	0,154 2	0,153 5	0,152 8	0,152 1	0,151 4
	0,89	0,150 7	0,150 0	0,149 3	0,148 6	0,147 9	0,147 2	0,146 5	0,145 7	0,145 0	0,144 3
	0,90	0,143 6	0,142 8	0,142 1	0,141 4	0,140 6	0,139 9	0,139 1	0,138 4	0,137 6	0,136 8
	0,91	0,136 1	0,135 3	0,134 5	0,133 8	0,133 0	0,132 2	0,131 4	0,130 6	0,129 8	0,129 0
	0,92	0,128 2	0,127 4	0,126 6	0,125 7	0,124 9	0,124 1	0,123 2	0,122 4	0,121 5	0,120 7
	0,93	0,119 8	0,118 9	0,118 1	0,117 2	0,116 3	0,115 4	0,114 5	0,113 6	0,112 7	0,111 8
	0,94	0,110 8	0,109 9	0,108 9	0,108 0	0,107 0	0,106 1	0,105 1	0,104 1	0,103 1	0,102 1
	0,95	0,101 1	0,100 1	0,099 0	0,098 0	0,096 9	0,095 9	0,094 8	0,093 7	0,092 6	0,091 5
	0,96	0,090 3	0,089 2	0,088 0	0,086 9	0,085 7	0,084 5	0,083 2	0,082 0	0,080 7	0,079 5
	0,97	0,078 2	0,076 8	0,075 5	0,074 1	0,072 7	0,071 3	0,069 9	0,068 4	0,066 9	0,065 3
	0,98	0,063 8	0,062 1	0,060 5	0,058 8	0,057 0	0,055 2	0,053 3	0,051 4	0,049 4	0,047 3
	0,99	0,045 1	0,042 7	0,040 3	0,037 7	0,034 9	0,031 8	0,028 5	0,024 7	0,020 1	0,014 2
	1,00	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0

NOTE For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Annex G
(normative)

Single sampling plans of Form *p**

Table G.1 — Maximum allowable values, *p, of the estimated process fraction nonconforming for sample sizes 3 and 4: *s*–method**

Inspection severity	Sample size code letter	Acceptance quality limit (in percent nonconforming)					
		1,0	1,5	2,5	4,0	6,5	10
		$p*$	$p*$	$p*$	$p*$	$p*$	$p*$
Tightened	B	—	—	—	—	0,192 5	0,255 0
	C	—	—	—	0,086 0	—	—
Normal	B	—	—	—	0,192 5	0,255 0	0,304 7
	C	—	—	0,086 0	—	—	—
Reduced	B—D	—	0,192 5	0,216 7	0,255 0	0,304 7	0,487 9
	E	0,086 0	—	—	—	—	—

Annex H
(normative)

Values of c_U for upper control limit on the sample standard deviation

Table H.1 — Values of c_U for upper control limit on the sample standard deviation

Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U
3	2,296 8	27	1,361 6	51	1,260 0	82	1,203 9	124	1,165 2	213	1,125 6
4	2,064 7	28	1,354 8	52	1,257 4	83	1,202 6	125	1,164 5	214	1,125 3
5	1,924 1	29	1,348 4	53	1,254 9	84	1,201 4	126	1,163 8	233	1,120 0
6	1,827 3	30	1,342 2	54	1,252 5	85	1,200 2	127	1,163 2	239	1,118 5
7	1,755 5	31	1,336 4	55	1,250 1	88	1,196 7	131	1,160 6	244	1,117 3
8	1,699 5	32	1,330 9	57	1,245 6	89	1,195 5	132	1,160 0	247	1,116 5
9	1,654 3	33	1,325 7	58	1,243 4	90	1,194 4	134	1,158 8	260	1,113 6
10	1,616 8	34	1,320 6	60	1,239 2	92	1,192 3	137	1,157 0	262	1,113 1
11	1,585 0	35	1,315 9	61	1,237 2	93	1,191 2	142	1,154 2	277	1,110 0
12	1,557 7	36	1,311 3	63	1,233 3	94	1,190 2	143	1,153 7	293	1,106 9
13	1,533 8	37	1,306 9	64	1,231 4	96	1,188 1	149	1,150 5	298	1,106 0
14	1,512 8	38	1,302 7	65	1,229 6	99	1,185 2	150	1,150 0	312	1,103 6
15	1,494 0	39	1,298 6	66	1,227 8	101	1,183 3	155	1,147 5	320	1,102 3
16	1,477 1	40	1,294 7	68	1,224 3	102	1,182 4	159	1,145 6	323	1,101 8
17	1,461 9	41	1,291 0	69	1,222 7	105	1,179 8	169	1,141 2	332	1,100 4
18	1,448 0	42	1,287 4	71	1,219 4	108	1,177 2	170	1,140 8	348	1,098 0
19	1,435 3	43	1,283 9	72	1,217 9	110	1,175 5	171	1,140 4	362	1,096 1
20	1,423 6	44	1,280 6	73	1,216 3	111	1,174 7	178	1,137 5	395	1,092 0
21	1,412 8	45	1,277 3	74	1,214 8	112	1,173 9	186	1,134 5	398	1,091 6
22	1,402 7	46	1,274 2	75	1,213 4	115	1,171 6	187	1,134 1	424	1,088 7
23	1,393 4	47	1,271 2	76	1,211 9	116	1,170 9	189	1,133 4	438	1,087 3
24	1,384 7	48	1,268 3	78	1,209 1	117	1,170 1	201	1,129 3	498	1,081 8
25	1,376 5	49	1,265 4	79	1,207 8	120	1,168 0	202	1,129 0	541	1,078 5
26	1,368 8	50	1,262 7	81	1,205 2	122	1,166 6	207	1,127 4		

NOTE Table entries are $\sqrt{\chi^2_{n-1,\gamma} \ / \ (n-1)}$ where $\chi^2_{n-1,\gamma}$ is the γ – fractile of the chi-squared distribution with n–1 degrees of freedom and $\gamma = 0{,}95^{0{,}1} = 0{,}994\ 884$.

Annex I
(normative)

Supplementary acceptability constants for qualifying towards reduced inspection

Table I.1 — Supplementary acceptability constants for qualifying towards reduced inspection

Sample size code letter	AQL (%)	Acceptability constant for AQL that is one step tighter		
		s–method		σ–method
		k	p^* (%)	k
B	4,0	1,114	8,502	0,918
C	2,5	1,409	3,041	1,325
D	1,5	1,601	*n/a*	1,562
E	1,0	1,825	*n/a*	1,752
F	0,65	2,029	*n/a*	2,013
G	0,40	2,209	*n/a*	2,161
H	0,25	2,390	*n/a*	2,379
J	0,15	2,530	*n/a*	2,523
K	0,10	2,689	*n/a*	2,667
L	0,065	2,857	*n/a*	2,847
M	0,040	2,995	*n/a*	2,972
N	0,025	3,143	*n/a*	3,131
P	0,015	3,254	*n/a*	3,246
Q	0,010	3,385	*n/a*	3,382
R	0,010	3,449	*n/a*	3,446
NOTE For this part of ISO 3951, n/a is "not applicable."				

NOTE These constants were calculated so that the probability of acceptance at the next lower AQL is the same as the probability of acceptance at the given AQL. For example, the normal inspection s–method sampling plan for sample size code letter B and AQL of 4,0 % is found from Table B.1 to be n = 3 and k = 0,950. This may be shown to have a probability of acceptance of 96,440 4 % at process quality level 4,0 %. The next smaller AQL is 2,5 %. The s–method form k acceptability constant that provides the same probability of acceptance 96,440 4 % with the same sample size n = 3 may be shown to be k = 1,114. Thus, in order to be acceptable at a one level tighter AQL, the sample mean needs to be at least 1,114 times the sample standard deviation within specification, rather than the 0,950 times the sample standard deviation needed to be considered merely acceptable.

Annex J
(normative)

Procedures for obtaining *s* and *σ*

J.1 Procedure for obtaining *s*

J.1.1 The estimate from a sample of the standard deviation of a population is generally denoted by the symbol *s*. Its value may be obtained from Formula (J.1).

$$s = \sqrt{\frac{\sum_{j=1}^{n}(x_j - \bar{x})^2}{n-1}} \tag{J.1}$$

where x_j is the value of the quality characteristic of the *j*th item in a sample of *n* articles, expressed as a decimal fraction, and $\bar{x}$ is the mean value of the x_j, i.e.

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j \tag{J.2}$$

J.1.2 Formula (J.1) for *s* is not recommended for the purpose of computation, as it tends to introduce an unnecessary amount of rounding error. An equivalent but computationally better formula is

$$s = \sqrt{\frac{n\sum_{j=1}^{n} x_j^2 - (\sum_{j=1}^{n} x_j)^2}{n(n-1)}} \tag{J.3}$$

J.1.3 If the variability is very small relative to the mean, i.e. *s* is very small in comparison with $\bar{x}$ Formula (J.3) can be improved upon still further by subtracting a suitable arbitrary constant, *a*, from all the values of before computing *s*, i.e.

$$s = \sqrt{\frac{n\sum_{j=1}^{n}(x_j - a)^2 - \left[\sum_{j=1}^{n}(x_j - a)\right]^2}{n(n-1)}} \tag{J.4}$$

J.1.4 Many pocket calculators have a standard deviation function key. Unfortunately, sometimes the sample size, *n*, is used by the machine in the denominator of Formula (J.1) instead of $n - 1$. If it is planned to use a calculator function or a computer program, it is important to check that the formula used by the machine is equivalent to Formula (J.1). A simple check is to find the standard deviation of the three numbers 0, 1, and 2. The sample size *n* is 3, the sample mean is 1, the deviations from the mean are −1, 0, and 1, the squares of the deviations are 1, 0, and 1, the sum of squares of the deviations is 2. So from Formula (J.1), we have

$$s = \sqrt{\frac{2}{2}} = \sqrt{1} = 1 \tag{J.5}$$

If the computer or calculator is erroneously using n instead of $n - 1$ in the denominator, then the result of the calculation will be

$$s = \sqrt{\frac{2}{3}} = 0{,}816\ 5$$

Use of n in the denominator must be avoided, for otherwise the acceptance criterion is weakened and the AOQL protection to the consumer is lost.

NOTE It is instructive to work through the use of Formula (J.3) for this example. It is found that

$$s = \sqrt{\frac{3 \times (0^2 + 1^2 + 2^2) - (0 + 1 + 2)^2}{3 \times (3 - 1)}} = \sqrt{\frac{3 \times (0 + 1 + 4) - 3^2}{3 \times 2}} = \sqrt{\frac{3 \times 5 - 9}{6}} = \sqrt{\frac{6}{6}} = 1$$

as before.

J.2 Procedure for obtaining σ

J.2.1 If it appears from the control chart that the value of s is in control, σ may be presumed to be the weighted root mean square of s given by the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m} (n_i - 1) s_i^2}{\sum_{i=1}^{m} (n_i - 1)}} \tag{J.6}$$

where

m is the number of lots;

n_i is the sample size from the ith lot;

s_i is the sample standard deviation from the ith lot.

J.2.2 If the sample sizes from each of the lots are equal, then Formula (J.6) simplifies to

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m} s_i^2}{m}} \tag{J.7}$$

Annex K
(informative)

Consumer's risk qualities

K.1 For a given sampling plan, the consumer's risk quality is the process quality for which the probability of accepting a given lot is 10 %.

K.2 For the s–method, the consumer's risk quality is the solution in p to the equation $F_{n-1,\sqrt{n}K_p}(\sqrt{n}k) = 0{,}90$, where n is the sample size, k is the s–method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $F_{n-1,\sqrt{n}K_p}(.)$ is the distribution function of the non-central t-distribution with $n-1$ degrees of freedom and non-centrality parameter $\sqrt{n}K_p$.

K.3 Consumer's risk qualities for the s–method plans of this part of ISO 3951 are given in Tables K.1, K.3, and K.5 for normal, tightened, and reduced inspection, respectively.

K.4 For the σ–method, the consumer's risk quality is given by the formula $\Phi\left[(1{,}2816 / \sqrt{n}) - k\right]$, where n is the sample size, k is the σ–method acceptability constant, and $\Phi(.)$ is the distribution function of the standard normal distribution.

K.5 Consumer's risk qualities for the σ–method plans of this part of ISO 3951 are given in Tables K.2, K.4, and K.6 for normal, tightened, and reduced inspection, respectively.

Table K.1 — Consumer's risk quality (in percent) for normal inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	53,0	52,3	56,4
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	39,5	36,5	39,9	54,1
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	26,4	24,5	27,1	41,4	51,2
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	16,7	15,8	17,7	27,8	36,8	44,8
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	11,7	10,7	11,8	18,7	24,8	30,7	41,4
G	↓	↓	↓	↓	↓	↓	↓	↓	7,37	6,97	7,73	12,2	16,2	20,0	27,6	34,5
H	↓	↓	↓	↓	↓	↓	↓	4,96	4,54	5,01	7,96	10,7	13,1	18,0	22,6	29,4
J	↓	↓	↓	↓	↓	↓	3,11	2,86	3,18	5,09	6,78	8,41	11,5	14,5	18,7	24,3
K	↓	↓	↓	↓	↓	2,01	1,85	2,05	3,27	4,39	5,45	7,46	9,39	12,2	15,8	22,0
L	↓	↓	↓	↓	1,26	1,16	1,29	2,06	2,78	3,43	4,72	5,94	7,71	10,0	13,9	↑
M	↓	↓	↓	0,812	0,743	0,826	1,33	1,77	2,19	3,02	3,79	4,91	6,39	8,91	↑	↑
N	↓	↓	0,515	0,471	0,521	0,830	1,12	1,39	1,91	2,40	3,12	4,05	5,64	↑	↑	↑
P	↓	0,323	0,296	0,328	0,521	0,705	0,873	1,19	1,50	1,95	2,54	3,53	↑	↑	↑	↑
Q	0,207	0,190	0,211	0,336	0,453	0,562	0,766	0,968	1,26	1,63	2,27	↑	↑	↑	↑	↑
R	0,119	0,132	0,209	0,284	0,352	0,481	0,605	0,786	1,02	1,42	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.2 — Consumer's risk quality (in percent) for normal inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	51,2	52,8	62,7
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	35,4	35,5	40,2	58,1
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	22,2	23,7	27,3	42,2	55,3
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,0	15,4	16,7	26,4	37,9	47,2
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,2	10,3	11,4	18,1	24,6	32,6	43,0
G	↓	↓	↓	↓	↓	↓	↓	↓	7,59	6,59	7,34	11,8	15,9	20,3	29,1	35,8
H	↓	↓	↓	↓	↓	↓	↓	4,50	4,30	4,85	7,85	10,7	13,2	19,2	23,7	30,2
J	↓	↓	↓	↓	↓	↓	2,76	2,77	3,12	5,07	6,79	8,33	12,0	15,1	19,4	24,9
K	↓	↓	↓	↓	↓	1,98	1,80	2,04	3,18	4,39	5,45	7,54	9,76	12,6	16,2	22,3
L	↓	↓	↓	↓	1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,16	7,95	10,2	14,2	↑
M	↓	↓	↓	0,840	0,738	0,807	1,32	1,78	2,18	3,02	3,94	5,05	6,55	9,06	↑	↑
N	↓	↓	0,510	0,469	0,518	0,821	1,11	1,38	1,91	2,48	3,20	4,13	5,73	↑	↑	↑
P	↓	0,311	0,284	0,317	0,523	0,696	0,865	1,19	1,55	2,00	2,59	3,59	↑	↑	↑	↑
Q	0,193	0,184	0,206	0,330	0,448	0,562	0,768	0,990	1,29	1,66	2,30	↑	↑	↑	↑	↑
R	0,116	0,131	0,204	0,282	0,352	0,480	0,616	0,803	1,04	1,45	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.3 — Consumer's risk quality (in percent) for tightened inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	53,0	52,3
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	39,5	36,5	39,9
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	26,4	24,5	27,1	41,4
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	16,7	15,8	17,7	27,8	36,8
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	11,7	10,7	11,8	18,7	24,8	30,7
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	7,37	6,97	7,73	12,2	16,2	20,0	27,6
H	↓	↓	↓	↓	↓	↓	↓	↓	4,96	4,54	5,01	7,96	10,7	13,1	18,0	24,9
J	↓	↓	↓	↓	↓	↓	↓	3,11	2,86	3,18	5,09	6,78	8,41	11,5	15,9	21,5
K	↓	↓	↓	↓	↓	↓	2,01	1,85	2,05	3,27	4,39	5,45	7,46	10,3	14,0	19,3
L	↓	↓	↓	↓	↓	1,26	1,16	1,29	2,06	2,78	3,43	4,72	6,52	8,85	12,2	↑
M	↓	↓	↓	↓	0,812	0,743	0,826	1,33	1,77	2,19	3,02	4,17	5,68	7,85	↑	↑
N	↓	↓	↓	0,515	0,471	0,521	0,830	1,12	1,39	1,91	2,64	3,58	4,97	↑	↑	↑
P	↓	↓	0,323	0,296	0,328	0,521	0,705	0,873	1,19	1,66	2,25	3,11	↑	↑	↑	↑
Q	↓	0,207	0,190	0,211	0,336	0,453	0,562	0,766	1,06	1,44	2,00	↑	↑	↑	↑	↑
R	0,116	0,119	0,132	0,209	0,284	0,352	0,481	0,666	0,906	1,25	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.4 — Consumer's risk quality (in percent) for tightened inspection: σ–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	51,2	52,8
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	35,4	35,5	40,2
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	22,2	23,7	27,3	42,2
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,0	15,4	16,7	26,4	37,9
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,2	10,3	11,4	18,1	24,6	32,6
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	7,59	6,59	7,34	11,8	15,9	20,3	29,1
H	↓	↓	↓	↓	↓	↓	↓	↓	4,50	4,30	4,85	7,85	10,7	13,2	19,2	26,0
J	↓	↓	↓	↓	↓	↓	↓	2,76	2,77	3,12	5,07	6,79	8,33	12,0	16,7	22,1
K	↓	↓	↓	↓	↓	↓	1,98	1,80	2,04	3,18	4,39	5,45	7,54	10,7	14,4	19,7
L	↓	↓	↓	↓	↓	1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,75	9,10	12,5	↑
M	↓	↓	↓	↓	0,840	0,738	0,807	1,32	1,78	2,18	3,02	4,31	5,83	7,99	↑	↑
N	↓	↓	↓	0,510	0,469	0,518	0,821	1,11	1,38	1,91	2,72	3,67	5,05	↑	↑	↑
P	↓	↓	0,311	0,284	0,317	0,507	0,696	0,865	1,19	1,71	2,30	3,16	↑	↑	↑	↑
Q	↓	0,193	0,184	0,206	0,330	0,448	0,562	0,768	1,10	1,47	2,03	↑	↑	↑	↑	↑
R	0,115	0,116	0,1314	0,204	0,282	0,352	0,480	0,684	0,924	1,27	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.5 — Consumer's risk quality (in percent) for reduced inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	53,0	49,2	52,3	56,4	61,1
E										↓	39,5	34,1	36,5	39,9	54,1	60,8
F									↓	26,4	23,9	24,5	27,1	41,4	51,2	59,4
G								↓	16,7	15,8	15,8	17,7	27,8	36,8	44,8	55,7
H							↓	11,7	10,5	10,7	11,8	18,7	24,8	30,7	41,4	47,2
J						↓	7,37	6,85	6,97	7,73	12,2	16,2	20,0	27,6	31,0	37,8
K					↓	4,96	4,48	4,54	5,01	7,96	10,7	13,1	18,0	20,3	24,9	29,4
L				↓	3,11	2,84	2,86	3,18	5,09	6,78	8,41	11,5	13,0	15,9	18,7	↑
M			↓	2,01	1,82	1,85	2,05	3,27	4,39	5,45	7,46	8,45	10,3	12,2	↑	
N		↓	1,26	1,16	1,16	1,29	2,06	2,78	3,43	4,72	5,31	6,52	7,71	↑		
P	↓	0,812	0,745	0,743	0,826	1,33	1,77	2,19	3,02	3,40	4,17	4,91	↑			
Q	0,515	0,473	0,471	0,521	0,830	1,12	1,39	1,91	2,15	2,64	3,12	↑				
R	0,297	0,296	0,328	0,521	0,705	0,873	1,19	1,35	1,66	1,95	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.6 — Consumer's risk quality (in percent) for reduced inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	51,2	48,5	52,8	62,7	63,3
E										↓	35,4	31,8	35,5	40,2	58,1	62,8
F									↓	22,2	22,9	23,7	27,3	42,2	55,3	61,1
G								↓	17,0	14,5	15,4	16,7	26,4	37,9	47,2	56,8
H							↓	10,2	9,61	10,3	11,4	18,1	24,6	32,6	43,0	48,5
J						↓	7,59	6,71	6,59	7,34	11,8	15,9	20,3	29,1	32,5	39,0
K					↓	4,50	4,30	4,30	4,85	7,85	10,7	13,2	19,2	21,4	26,0	30,2
L				↓	2,76	2,69	2,77	3,12	5,07	6,79	8,33	12,0	13,7	16,7	19,4	↑
M			↓	1,98	1,73	1,80	2,04	3,18	4,39	5,45	7,54	8,86	10,7	12,6	↑	
N		↓	1,20	1,18	1,15	1,25	2,05	2,78	3,43	4,72	5,54	6,75	7,95	↑		
P	↓	0,840	0,743	0,738	0,807	1,32	1,78	2,18	3,02	3,47	4,31	5,05	↑			
Q	0,510	0,467	0,469	0,518	0,821	1,11	1,38	1,91	2,16	2,72	3,20	↑				
R	0,292	0,284	0,317	0,523	0,696	0,865	1,19	1,36	1,71	2,00	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Annex L
(informative)

Producer's risks

L.1 The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL, i.e. 1 minus the probability of accepting a given lot when the process fraction nonconforming is equal to the AQL.

L.2 For the *s*–method, the producer's risk is given by the formula $F_{n-1,\sqrt{n}K_p}(\sqrt{n}k)$, where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the *s*–method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $F_{n-1,\sqrt{n}K_p}(.)$ is the distribution function of the non-central *t*-distribution with degrees of freedom $n-1$ and non-centrality parameter $\sqrt{n}K_p$.

L.3 Producer's risks for the *s*–method plans of this part of ISO 3951 are given in Tables L.1, L.3, and L.5 for normal, tightened, and reduced inspection, respectively.

L.4 For the σ–method, the producer's risk is given by the formula $\Phi\left[\sqrt{n}(k-K_p)\right]$, where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the σ–method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $\Phi(.)$ is the distribution function of the standard normal distribution.

L.5 Producer's risks for the σ–method plans of this part of ISO 3951 are given in Tables L.2, L.4, and L.6 for normal, tightened, and reduced inspection, respectively.

Table L.1 — Producer's risk (in percent) for normal inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B													↓	10,8	7,46	8,93
C												↓	12,2	8,00	10,8	5,82
D											↓	10,8	7,52	10,3	8,74	2,50
E										↓	10,3	6,88	10,4	9,07	4,62	3,18
F									↓	10,8	7,12	8,54	8,14	3,77	3,34	0,908
G								↓	9,81	7,62	9,99	7,49	3,94	3,35	1,45	1,10
H							↓	9,88	6,98	9,99	7,95	3,37	3,07	1,21	1,30	0,853
J						↓	8,91	6,61	9,63	8,64	3,91	2,71	1,26	1,28	1,27	1,13
K					↓	9,16	5,79	9,08	7,65	3,99	3,14	0,891	1,12	1,01	1,48	0,568
L				↓	9,45	6,29	8,16	7,54	3,78	3,51	1,24	0,891	1,08	1,37	1,05	↑
M			↓	9,01	6,54	8,99	6,77	3,51	3,12	1,39	1,19	0,685	1,23	0,787	↑	
N		↓	8,76	6,26	9,48	7,30	2,97	2,98	1,20	1,43	1,07	0,803	0,741	↑		
P	↓	8,09	6,12	9,15	7,88	3,60	2,55	1,18	1,27	1,42	1,44	0,462	↑			
Q	8,47	5,32	8,68	7,20	3,74	2,93	0,806	1,10	1,07	1,66	0,759	↑				
R	6,00	7,90	7,07	3,52	3,35	1,14	0,821	1,05	1,42	1,18	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.2 — Producer's risk (in percent) for normal inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B													↓	3,57	2,96	6,72
C												↓	7,17	3,59	6,06	4,54
D											↓	6,33	3,89	6,37	4,81	2,86
E										↓	7,17	3,94	6,29	4,62	2,81	2,74
F									↓	7,65	4,32	5,42	4,66	1,89	2,80	0,865
G								↓	7,44	4,96	6,87	4,66	2,04	2,09	1,41	1,15
H							↓	7,47	4,68	7,35	5,48	1,98	1,86	1,26	1,38	0,871
J						↓	6,69	4,70	7,38	6,40	2,56	1,59	1,05	1,25	1,32	1,24
K					↓	7,32	4,16	7,17	5,56	2,74	2,10	0,572	1,08	1,06	1,58	0,602
L				↓	7,64	4,82	6,30	5,80	2,67	2,48	0,788	0,854	1,09	1,39	1,07	↑
M			↓	7,52	5,16	7,26	5,29	2,56	2,19	0,933	1,17	0,682	1,28	0,829	↑	
N		↓	7,30	5,02	7,95	5,82	2,04	2,12	0,844	1,36	1,07	0,808	0,774	↑		
P	↓	6,70	4,77	7,55	6,30	2,64	1,82	0,832	1,23	1,42	1,46	0,481	↑			
Q	7,06	4,16	7,25	5,85	2,84	2,26	0,578	1,02	1,07	1,69	0,776	↑				
R	4,89	6,71	5,76	2,73	2,68	0,830	0,738	1,04	1,43	1,20	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.3 — Producer's risk (in percent) for tightened inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B														↓	17,6	14,7
C													↓	19,0	16,7	21,7
D												↓	18,1	15,7	22,7	18,6
E											↓	16,2	15,8	22,4	20,7	13,4
F										↓	17,3	13,9	20,6	18,7	13,2	13,0
G									↓	17,2	15,4	19,7	18,8	13,1	15,0	9,41
H								↓	16,9	15,7	20,8	16,8	12,7	13,6	10,2	5,66
J							↓	16,4	14,9	22,1	18,8	11,4	13,8	9,92	7,33	5,18
K						↓	15,1	14,3	20,7	19,0	12,5	11,6	9,13	6,21	6,13	4,11
L					↓	16,1	13,1	20,5	18,4	13,7	13,8	7,85	6,26	5,82	6,19	↑
M				↓	16,5	14,2	18,7	18,1	12,6	14,8	9,52	4,65	5,34	5,00	↑	
N			↓	15,9	15,0	20,5	16,3	12,3	13,8	10,7	6,36	3,84	4,76	↑		
P		↓	15,7	14,5	21,8	18,2	11,2	13,9	10,1	7,75	6,00	3,38	↑			
Q	↓	14,5	13,9	20,5	18,7	12,4	11,5	9,18	6,35	6,64	4,86	↑				
R	12,4	12,9	20,4	18,1	13,5	13,8	7,80	6,33	6,01	6,68	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.4 — Producer's risk (in percent) for tightened inspection: σ–method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B														↓	8,17	7,75
C													↓	13,6	10,2	15,2
D												↓	13,4	10,5	17,2	12,6
E											↓	12,5	11,5	17,4	14,5	9,78
F										↓	14,1	10,2	16,5	13,9	9,18	11,3
G									↓	14,3	12,0	16,1	14,7	9,35	11,8	8,77
H								↓	14,4	12,6	17,7	13,4	9,68	10,6	9,80	5,73
J							↓	14,1	12,3	19,3	15,9	8,94	10,9	8,86	7,29	5,19
K						↓	13,0	12,0	18,3	16,2	10,2	9,37	7,54	5,99	6,15	4,19
L					↓	14,1	11,1	18,2	16,1	11,5	11,7	6,29	6,07	5,79	6,10	↑
M				↓	14,7	12,3	16,7	16,0	10,8	12,7	7,96	4,50	5,31	4,97	↑	
N			↓	14,2	13,3	18,7	14,3	10,4	11,9	9,26	6,18	3,79	4,73	↑		
P		↓	14,2	12,7	20,1	16,3	9,49	12,1	8,77	7,61	5,94	3,40	↑			
Q	↓	13,0	12,3	18,8	17,0	10,7	10,1	8,07	6,26	6,54	4,84	↑				
R	11,0	11,4	18,9	16,5	12,0	12,4	6,74	6,24	5,96	6,67	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.5 — Producer's risk (in percent) for reduced inspection: *s*–method

Code letter	Acceptance quality limit (in percent nonconforming)														
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	3,77	2,70	3,29	3,91	0,257
E										↓	4,92	2,65	3,69	4,52	2,05	0,041
F									↓	4,28	3,18	3,08	4,33	3,41	0,571	0,159
G								↓	3,30	3,37	3,30	3,98	3,67	1,18	0,655	0,017
H							↓	3,38	2,67	3,22	3,80	2,87	0,915	0,544	0,082	0,214
J						↓	2,67	2,70	3,10	4,34	3,23	0,838	0,573	0,112	0,338	0,290
K					↓	2,97	2,28	2,84	3,77	3,20	0,978	0,421	0,092	0,256	0,379	0,853
L				↓	2,81	2,45	2,38	3,69	3,08	1,03	0,557	0,072	0,273	0,370	1,27	↑
M			↓	2,49	2,35	2,53	3,09	2,68	0,849	0,581	0,087	0,184	0,325	1,01	↑	
N		↓	2,36	2,26	2,57	3,40	2,34	0,822	0,503	0,107	0,272	0,235	1,08	↑		
P	↓	2,09	2,14	2,36	3,56	2,66	0,631	0,457	0,084	0,309	0,333	0,685	↑			
Q	2,19	1,89	2,30	3,30	2,65	0,743	0,349	0,074	0,256	0,419	1,07	↑				
R	2,09	2,01	3,25	2,49	0,838	0,459	0,052	0,244	0,363	1,42	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.6 — Producer's risk (in percent) for reduced inspection: *σ*–method

Code letter	Acceptance quality limit (in percent nonconforming)														
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	0,570	0,519	0,913	2,87	0,367
E										↓	1,80	0,601	1,16	1,88	1,55	0,065
F									↓	1,56	1,18	1,14	2,08	1,42	0,772	0,217
G								↓	1,72	1,35	1,49	1,68	1,27	0,561	0,560	0,021
H							↓	1,57	1,12	1,54	1,90	1,15	0,307	0,439	0,081	0,222
J						↓	1,58	1,46	1,56	2,36	1,59	0,278	0,263	0,116	0,359	0,323
K					↓	1,64	1,21	1,51	2,23	1,80	0,438	0,173	0,105	0,275	0,443	0,871
L				↓	1,55	1,37	1,36	2,36	1,88	0,524	0,234	0,054	0,278	0,418	1,32	↑
M			↓	1,59	1,36	1,55	2,06	1,57	0,447	0,293	0,042	0,185	0,326	1,06	↑	
N		↓	1,47	1,53	1,72	2,25	1,49	0,456	0,266	0,047	0,261	0,235	1,09	↑		
P	↓	1,46	1,44	1,61	2,50	1,82	0,361	0,240	0,039	0,230	0,327	0,682	↑			
Q	1,51	1,26	1,62	2,43	1,84	0,407	0,181	0,038	0,158	0,412	1,07	↑				
R	1,44	1,31	2,31	1,67	0,498	0,258	0,026	0,161	0,368	1,42	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Annex M
(informative)

Operating characteristics for the σ-method

M.1 Formula for probability of acceptance

The exact probability of lot acceptance for a single specification limit at process fraction nonconforming, p, when the process standard deviation is known is given by Formula (M.1),

$$P_a = \Phi\left[\sqrt{n}(K_p - k)\right] \tag{M.1}$$

where

$\Phi(.)$ is the standard normal distribution function;

n is the sample size;

K_p is the upper p-fractile of the standard normal distribution;

k is the σ–method acceptance constant.

M.2 Example

Consider the calculation of the probability of acceptance at a process quality of 2,5 % nonconforming for a σ–method plan with AQL of 1,0 % and sample size code letter M under normal inspection. Entering Table C.1 with sample size code letter M and AQL of 1,0 %, it is found that the sample size, n, is 39 and the acceptance constant, k, is 1,963. The process fraction nonconforming under consideration is P = 0,025 0, and from tables of the standard normal distribution, it is found that K_p = 1,960. Hence,

$$P_a = \Phi\left[\sqrt{39}(1{,}960 - 1{,}963)\right] = \Phi(-0{,}018\,7)$$

which, from standard normal distribution tables, yields P_a = 0,492 5.

M.3 Comparison with tabulated value for the s–method

It is instructive to observe that this probability of acceptance for the σ–method is very roughly in agreement with the corresponding probability of acceptance for the s–method. From the column of the table in Chart M for AQL of 1,0 %, it is seen that a process quality level of 2,43 %, i.e. P = 0,024 3, corresponds to a probability of acceptance of 50 %, i.e. to P_a = 0,500.

Annex N
(informative)

Estimating the process fraction nonconforming for sample sizes 3 and 4: *s*-method

N.1 General formula for sample size, *n*

The general formula for the estimator of the process fraction nonconforming beyond either of the specification limits when the process standard deviation is unknown is

$$\hat{p} = B_{(n-2)/2}\left\{\left[1 - Q\sqrt{n} / (n-1)\right] / 2\right\} \tag{N.1}$$

where

n is the sample size;

Q is the quality statistic;

$B_{(n-2)/2}(.)$ is the symmetric beta distribution function with both parameters equal to $(n - 2) / 2$.

N.2 Formula for sample size 3

When $n = 3$, the estimator becomes

$$\hat{p} = B_{1/2}\left[(1 - Q\sqrt{3} / 2)\ 2\right] \tag{N.2}$$

Now

$$B_{1/2}(x) = \begin{cases} 0 & \text{if } x < 0 \\ \int_0^x \frac{t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}}{B(\frac{1}{2},\frac{1}{2})} dt & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{N.3}$$

where

$$B(\tfrac{1}{2},\tfrac{1}{2}) = \Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{2}) / \Gamma(\tfrac{1}{2}+\tfrac{1}{2}) = \sqrt{\pi}\sqrt{\pi} / 1 = \pi$$

with $\Gamma(.)$ representing the gamma function. Writing $t = \sin^2 \theta$ Formula (N.3) becomes

$$B_{1/2}(x) = \begin{cases} 0 \\ \frac{2}{\pi}\int_0^{\arcsin(\sqrt{x})} d\theta \\ 1 \end{cases} = \begin{cases} 0 & \text{if } x < 0 \\ \frac{2}{\pi}\arcsin(\sqrt{x}) & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{N.4}$$

Hence, substituting Formula (N.4) in Formula (N.2),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2 / \sqrt{3} \\ \frac{2}{\pi}\arcsin\left[\sqrt{(1 - Q\sqrt{3} / 2) / 2}\right] & \text{if } -2 / \sqrt{3} \le Q \le 2 / \sqrt{3} \\ 1 & \text{if } Q < -2 / \sqrt{3} \end{cases} \tag{N.5}$$

This is the quantity tabulated in Annex F.

N.3 Formula for sample size 4

When n = 4, the estimator becomes

$$\hat{p} = B_1\left[\frac{1}{2}\left(1 - \frac{2}{3}Q\right)\right] = B_1\left[0.5 - Q / 3\right] \tag{N.6}$$

Now

$$B_1(x) = \begin{bmatrix} 0 & \text{if } x < 0 \\ \int_0^x \frac{dt}{B(1,1)} & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{bmatrix} \tag{N.7}$$

where $B(1,1) = \Gamma(1)\Gamma(1) / \Gamma(1 + 1) = 1$. Formula (N.7) can therefore be written as

$$B_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{N.8}$$

Hence, substituting Formula (N.8) in Formula (N.6),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 1{,}5 \\ 0{,}5 - Q / 3 & \text{if } -1{,}5 \leq Q \leq 1{,}5 \\ 1 & \text{if } Q < -1{,}5 \end{cases}$$

Annex O
(normative)

Accommodating measurement variability

O.1 General

The master tables of this part of ISO 3951 are based on the assumption that the true values of the quality characteristic, X, of the items in the lots are normally distributed with unknown process mean, μ, and either known or unknown process standard deviation, σ; the assumption is also made that X can be measured without measurement error, i.e. that the measurement of an item with the true value, x_i, results in the value x_i. This annex explains how these master tables may be used in the presence of measurement error.

In the case of measurement error, the measured value of an item with true value, x_i, will differ from x_i. It is assumed that

— the measurement method is unbiased, i.e. the expectation of the measurement error is zero;

— measurement error inflates the perceived process variation and is independent of the actual process standard deviation;

— measurement error is normally distributed with known or unknown measurement standard deviation, σ_{m}.

It follows that the distribution of the measured values is a normal distribution with mean μ, and standard deviation

$$\sigma_{\text{total}} = \sqrt{\sigma^2 + \sigma_m^2} \tag{O.1}$$

Note that σ_{total} is always larger than σ if measurement error exists.

If it is known that $\sigma_{\mathrm{m}} < \sigma / 10$, i.e. the ratio $\gamma = \sigma_m / \sigma$ of measurement standard deviation to process standard deviation is less than 10 %, the total standard deviation is

$$\sigma_{\text{total}} < \sqrt{\sigma^2 + (0{,}1\sigma)^2} = \sigma\sqrt{1 + 0{,}01} = 1{,}005\sigma \tag{O.2}$$

i.e. the standard deviation is increased by less than 0,5 %, which is negligible and hence, the sampling plans do not need to be adjusted for measurement error.

In cases where $\sigma_{\mathrm{m}} \geq 0{,}1\sigma$, the sampling plans of this part of ISO 3951 shall be used with the following adjustments.

1. Increase the sample size, n, in order to compensate for the perceived inflated variability, but do not alter the acceptability constant, k or p^*.

2. When the process standard deviation, σ, is known, use σ in calculating the test statistic, $\overline{x} \pm k\sigma$ or $\hat{p}$; otherwise, use an estimate s of σ in calculating the test statistic $\overline{x} \pm ks$ or $\hat{p}$.

Further details are given in the following sub-clauses for three distinct cases.

O.2 Process standard deviation σ and measurement standard deviation σ_m both known

1. Increase the sample size, n, of the sampling plan to

$$n^* = n(1 + \gamma^2) \tag{O.3}$$

2. Use the process standard deviation, σ, in calculating the test statistic, $\bar{x} \pm k\sigma$ or $\hat{p}$.

O.3 Process standard deviation σ unknown but measurement standard deviation σ_m known

1. Increase the sample size, n, of the sampling plan to

$$n^* = n(1 + \tilde{\gamma}^2) \tag{O.4}$$

where $\tilde{\gamma}$ is an estimated upper bound of $\gamma = \sigma_m / \sigma$.

NOTE As $\tilde{\gamma}$ increases, the operating characteristic curve of the sampling plan turns clockwise around the indifference quality point ($p_{50\,\%}$, 0,5), i.e. the point where the probability of acceptance of the lot is 50 %. If γ is overestimated ($\tilde{\gamma}$ larger than γ), the sampling plan is better than required, i.e. its probabilities of acceptance are larger than required for $P > p_{50\,\%}$ and smaller than required for $P > p_{50\,\%}$. Hence, overestimation of γ ensures a sampling plan that is better than required.

2. Use the estimate

$$s^* = \sqrt{s^2 - \sigma_m^2} \tag{O.5}$$

of the process standard deviation instead of s in calculating the test statistic $\bar{x} \pm ks$ or $\hat{p}$.

If $s^2 - \sigma_m^2 < 0$, use $s^*=0$.

O.4 Process standard deviation σ and measurement standard deviation σ_m both unknown

Increase the sample size, n, in accordance with Formula (O.4), perform duplicate (or multiple) measurements on each sampled item, and use the measurement results to estimate the process standard deviation separately from the measurement standard deviation, as shown below. Use this estimate instead of s in calculating the test statistic $\bar{x} \pm ks$ or $\hat{p}$.

Estimation of the process and measurement standard deviations.

We denote the jth measurement on the ith item by x_{ij}, the mean for the ith item by $\bar{x}_{i}$, and the overall mean by $\bar{x}_{..}$. The number of measurements for the ith item will be denoted by n_i. The total sum of squares of the measurements about their overall mean can be partitioned as follows:

$$\begin{aligned}
\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(x_{ij}-\bar{x}_{..}\right)^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(x_{ij}-\bar{x}_{i.}+\bar{x}_{i.}-\bar{x}_{..}\right)^2 \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}\left[(x_{ij}-\bar{x}_{i.})^2+(\bar{x}_{i.}-\bar{x}_{..})^2+2(x_{ij}-\bar{x}_{i.})(\bar{x}_{i.}-\bar{x}_{..})\right] \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2+2\sum_{i=1}^{n}(\bar{x}_{i.}-\bar{x}_{..})\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.}) \qquad (0.6)\\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2+\square \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2 \\
&= W+B
\end{aligned}$$

where

W is the within-items sum of squares;

B is the between-items sum of squares.

The expectations of these sums of squares are

$$E(W) = \sigma_m^2\sum_{i=1}^{n}\left(n_i-1\right) = \sigma_m^2\left(N-n\right) \qquad (0.7)$$

where $N = \sum_{i=1}^{n} n_i$ is the total number of observations, and

$$E(B) = \sigma_m^2\left(n-1\right)+(N-n)\sigma^2 \qquad (0.8)$$

Hence, σ_{m}^2 can be estimated by

$$\hat{\sigma}_m^2 = W / (N-n) \qquad (0.9)$$

and σ^2 can be estimated by

$$s^2 = \hat{\sigma}^2 = \left[B-(n-1)\hat{\sigma}_m^2\right] / (N-n) \qquad (0.10)$$

Example

A manufactured component has a dimension with an upper specification limit of 13,05 cm. The process standard deviation, σ, and measurement standard deviation, σ_{m}, are unknown, but from previous experience, it is known that the ratio $\sigma_{\mathrm{m}}/\sigma$ is greater than 0,1 but less than 0,2. Lots of size 1 000 of these components are to be inspected. Normal inspection is to be instituted with an AQL of 0,15 %.

From Table A.1, it is found that the sample size code letter is J. As only one specification limit is being controlled, Form k can be used; from Table B.1, the sampling plan for an AQL of 0,15 % in the absence of sampling error is $n = 23$, $k = 2{,}425$.

As σ_m / σ exceeds 0,1, it is necessary to adjust the sample size to allow for measurement uncertainty.

In the presence of the worst conceivable measurement error, the appropriate sample size (from Formula O.3) is given by

$$n^* = n(1 + \tilde{\gamma}^2) = 23\left(1 + (0{,}2)^2\right) = 23 \times 1{,}04 = 23{,}92$$

The sample size must be an integer so, in order to provide at least the required AQL protection, n^* is rounded *up* to $n^* = 24$. A random sample of 24 of the components is taken from the next lot, and, in order to be able to assess the measurement uncertainty, each component is measured twice. The results for the sample from the first lot are as follows:

Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}
1	12,997 2	12,999 7	6	13,023 1	13,021 9	11	12,956 2	12,962 1	16	12,957 8	12,952 7	21	13,000 9	12,999 3
2	12,984 8	12,973 1	7	12,993 0	12,993 7	12	12,988 6	12,986 7	17	12,976 5	12,967 4	22	13,003 4	12,994 5
3	12,964 6	12,963 0	8	12,958 9	12,943 9	13	13,007 1	13,008 3	18	12,999 1	13,001 0	23	12,965 1	12,962 5
4	12,954 3	12,953 9	9	12,958 9	12,952 4	14	12,978 7	12,973 8	19	13,002 9	13,006 7	24	12,986 5	12,985 2
5	12,976 3	12,980 2	10	13,015 0	13,016 4	15	12,927 4	0,927 7	20	12,968 8	12,976 2			

The accuracy of subsequent calculations can be improved by subtracting an arbitrary constant that reduces the number of significant figures. Denote the constant by c and set $c = 12{,}9$. The resulting values of $y_{ij} = x_{ij} - 12{,}9$ are

Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}
1	0,097 2	0,099 7	6	0,123 1	0,121 9	11	0,056 2	0,062 1	16	0,057 8	0,052 7	21	0,100 9	0,099 3
2	0,084 8	0,071 1	7	0,093 0	0,093 7	12	0,088 6	0,086 7	17	0,076 5	0,067 4	22	0,103 4	0,094 5
3	0,064 6	0,063 0	8	0,058 9	0,043 9	13	0,107 1	0,108 3	18	0,099 1	0,101 0	23	0,065 1	0,062 5
4	0,054 3	0,053 9	9	0,058 9	0,052 4	14	0,078 7	0,073 8	19	0,102 9	0,099 2	24	0,086 5	0,085 2
5	0,076 3	0,080 2	10	0,115 0	0,116 4	15	0,027 4	0,027 7	20	0,068 8	0,076 2			

The sum of the y_{ij} is $\sum_{i=1}^{24} \sum_{j=1}^{2} y_{ij} = 3{,}839\,9$

The sample mean value of y is $\bar{y} = 3{,}839\,9 / 48 = 0{,}079\,998$

Hence, the sample mean value of x is $\bar{x} = c + \bar{y} = 12{,}9 + 0{,}079\,998 = 12{,}979\,998$

The total sum of squares of y is $T = \sum_{i=1}^{24} \sum_{j=1}^{2} y_{ij}^2 = 0{,}332\,791\,15$

The total sum of squares, T, about the overall sample mean

$$= \sum_{i=1}^{24} \sum_{j=1}^{2} y_{ij}^2 - \sum_{i=1}^{24} \left[\left(\sum_{j=1}^{2} y_{ij} \right)^2 / 2 \right] \tag{O.11}$$

$= 0{,}332\,791\,15 - 0{,}307\,184\,00$

$= 0{,}025\,607\,15$

The within-items sum of squares, W, is given by

$$W = \sum_{i=1}^{24} \sum_{j=1}^{2} (y_{ij} - \bar{y}_{i.})^2$$

$$= \sum_{i=1}^{24} \sum_{j=1}^{2} y_{ij}^2 - \sum_{i=1}^{24} \left(\sum_{j=1}^{2} y_{ij} \right)^2 / 2 \qquad (O.12)$$

$$= 0{,}0{,}33279115 - 0{,}332\ 407\ 52$$

$$= 0{,}000\ 383\ 63$$

By subtraction, the between-item sum of squares, B, is given by

$$B = T - W$$

$$= 0{,}025\ 607\ 15 - 0{,}000\ 383\ 63 \qquad (O.13)$$

$$= 0{,}025\ 223\ 52$$

The measurement error variance is estimated as

$$\hat{\sigma}_m^2 = W / (N - n) = 0{,}000\ 383\ 63 / (48 - 24) = 0{,}000\ 015\ 984\ 6$$

The process variance is estimated as

$$s^2 = \hat{\sigma}^2 = \left[B - (n - 1)\hat{\sigma}_m^2 \right] / (N - n)$$

$$= \left[0{,}025\ 223\ 52 - 23 \times 0{,}000\ 015\ 984\ 6 \right] / (48 - 24)$$

$$= 0{,}024\ 855\ 87 / 24$$

$$= 0{,}001\ 035\ 66$$

so the process standard deviation is estimated as

$$s = \hat{\sigma} = \sqrt{0{,}001\ 035\ 66} = 0{,}032\ 182$$

U – 2,419 s = 13,05 – 2,425 × 0,032 182 = 12,972

As $\bar{x}$ = 12,980 > 12,972, the lot is not accepted.

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